Special Case $\ell = 0$

$r < a$ $\qquad$ $r > a$

$\hat{E}_z = A_1 J_0(k_{\perp 1} r) \qquad E_z = A_2 \cancel{K}_0(\gamma_2 r)$

$\hat{H}_z = B_1 J_0(k_{\perp 1} r) \qquad H_z = B_2 K_0(\gamma_2 r)$

Region I

$$E_\theta(a) = \frac{-i\omega\mu}{k_{\perp 1}^2} B_1 k_{\perp 1} J_0'(k_{\perp 1} a)$$

$$H_\theta(a) = \frac{i\omega\epsilon}{k_{\perp 1}^2} A_1 k_{\perp 1} J_0'(k_{\perp 1} a)$$

Region II

$$E_\theta(a) = \frac{i\omega\mu}{\gamma_2} B_2 K_0'(\gamma_2 a)$$

$$H_\theta(a) = -\frac{i\omega\epsilon}{\gamma_2} A_2 K_0'(\gamma_2 a)$$

TE & TM modes

TE $(H_r, 0, H_z) \quad (0, E_\theta, 0)$

TM $(0, H_\theta, 0) \quad (E_r, 0, E_z)$

TE $\quad H_z \rightarrow \quad B_1 J_0(k_{21} a) = B_2 K_0(\gamma_2 r)$

$E_\theta \rightarrow \quad -i\frac{\omega\mu}{k_{21}} B_1 J_0'(k_{21} a) = i\frac{\omega\mu}{\gamma_2} B_2 K_0'(\gamma_2 a)$

$$\boxed{\frac{-i\omega\mu}{k_{21}}\frac{J_0'(k_{21} a)}{J_0} = \frac{\omega\mu}{\gamma_2}\frac{K_0'(\gamma_2 a)}{K_0(\gamma_2 a)}}$$

TM $\quad E_z \rightarrow \quad A_1 J_0(k_{21} a) = A_2 K_0(\gamma_2 r)$

$H_\theta \rightarrow \quad \frac{i\omega\epsilon_1}{k_{21}} A_1 J_0'(k_{21} a) = -\frac{i\omega\epsilon_0}{\gamma_2} A_2 K_0'(\gamma_2 r)$

$$\boxed{\frac{\epsilon_1}{k_{21}}\frac{J_0'}{J_0} = -\frac{\epsilon_0}{\gamma_2}\frac{K_0'}{K_0}}$$