Gyrotropic

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon_\perp & i\epsilon_x & 0 \\ -i\epsilon_x & \epsilon_\perp & 0 \\ 0 & 0 & \epsilon_{\parallel} \end{bmatrix}$$

$$\left[\underline{k}\,\underline{k} - \underline{\underline{1}}\,k^2 + k_0^2\underline{\underline{\epsilon}}\right]\cdot\hat{\underline{E}} = 0$$

Special case: take k in $\hat{z}$ dir

$$k_0^2\begin{bmatrix} \epsilon_\perp & i\epsilon_x & 0 \\ -i\epsilon_x & \epsilon_\perp & 0 \\ 0 & 0 & \epsilon_{\parallel} \end{bmatrix} + \begin{bmatrix} -k_z^2 & 0 & 0 \\ 0 & -k_z^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\cdot\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

$E_x, E_y = 0$

$$k_0^2\begin{bmatrix} \epsilon_\perp & i\epsilon_x \\ -i\epsilon_x & \epsilon_\perp \end{bmatrix}$$

$$\begin{bmatrix} k_0^2\epsilon_\perp - k_z^2 & ik_0^2\epsilon_x \\ -ik_0^2\epsilon_x & k_0^2\epsilon_\perp - k_z^2 \end{bmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix} = 0$$

DISPERSION Rela

$$(k_0^2\epsilon_\perp - k_z^2)^2 - (k_0^2\epsilon_x)^2 = 0$$

$$\cancel{k_z^2 - k_0^2\epsilon_\perp = \pm k_0^2\epsilon_x}$$

$$k_z^2 = k_0^2(\epsilon_\perp \pm \epsilon_x)$$ two k_z^2 values

$$k_0^2\epsilon_\perp - k_z^2 = \pm k_0^2\epsilon_x \qquad k_z^2 = k_0^2(\epsilon_\perp \mp \epsilon_x)$$

$$\pm k_0^2\epsilon_x E_x + i k_0^2\epsilon_x E_y = 0$$

$$\boxed{\frac{E_y}{E_x} = \pm i}$$ circularly polarized

Faraday Rotation

Part #1 Ponderomotive force

Cold fluid Equations

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u}\cdot\nabla \underset{\sim}{u} = \frac{q}{m}\left(\underset{\sim}{E} + \frac{\underset{\sim}{u}\times \underset{\sim}{B}}{c}\right)$$

$$\underset{\sim}{E} = \underset{\sim}{E}_f + \underset{\sim}{E}_s \qquad \underset{\sim}{E}_f = \mathrm{Re}\left\{ \hat{\underset{\sim}{E}}_f(\underset{\sim}{x},t)\, e^{-i\omega_f t} \right\}$$

$$\underset{\sim}{u} = \underset{\sim}{u}_f + \underset{\sim}{u}_s \qquad \underset{\sim}{u}_f = \mathrm{Re}\left\{ \hat{\underset{\sim}{u}}_f(\underset{\sim}{x},t)\, e^{-i\omega_f t} \right\}$$

fast ~~field~~ flow is ~~as~~ the linear high frequency response

$$\frac{\partial \underset{\sim}{u}_f}{\partial t} = \frac{q}{m}\underset{\sim}{E}_f$$

Slow flow contains nonlinear terms

$$\frac{\partial u_s}{\partial t} = \frac{q}{m}\left(\underset{\sim}{E}_s\right) - \left\langle \underset{\sim}{u}_f\cdot\nabla \underset{\sim}{u}_f \right\rangle_{\omega_f} + \frac{q}{m}\left\langle \frac{\underset{\sim}{u}_f\times \underset{\sim}{B}_f}{c} \right\rangle_{\omega_f}$$

this means average ~~of~~ over fast time $T_f = \frac{2\pi}{\omega_f}$

Note $\quad \underset{\sim}{u}_f \cdot \nabla \underset{\sim}{u}_f = \nabla \frac{1}{2} \underset{\sim}{u}_f \cdot \underset{\sim}{u}_f - \underset{\sim}{u}_f \times \nabla \times \underset{\sim}{u}_f$

$$\frac{\partial \underset{\sim}{u}_s}{\partial t} = \frac{q}{m} \underset{\sim}{E}_s - \nabla \frac{1}{2} \langle \underset{\sim}{u}_f \cdot \underset{\sim}{u}_f \rangle_{\omega_f}$$

$$+ \left\langle \underset{\sim}{u}_f \times \left[\frac{q \underset{\sim}{B}_f}{mc} + \nabla \times \underset{\sim}{u}_f \right] \right\rangle_{\omega_f}$$

This term averages to zero

$$\frac{\partial \underset{\sim}{u}_f}{\partial t} = \frac{q}{m} \underset{\sim}{E}_f$$

$$-\frac{1}{c} \frac{\partial \underset{\sim}{B}_f}{\partial t} = \nabla \times \underset{\sim}{E}_f$$

$$\frac{\partial}{\partial t} \left[\frac{q \underset{\sim}{B}_f}{mc} + \nabla \times \underset{\sim}{u}_f \right] = \frac{q}{mc} \frac{\partial \underset{\sim}{B}_f}{\partial t} + \nabla \times \frac{\partial \underset{\sim}{u}_f}{\partial t}$$

$$= -\frac{q}{m} \nabla \times \underset{\sim}{E}_f + \nabla \times \frac{q}{m} \underset{\sim}{E}_f = 0$$

ponderomotive force

$$m \frac{\partial \underset{\sim}{u}_s}{\partial t} = q \underset{\sim}{E}_s + \underset{\sim}{F}_p$$

Ponderomotive potential

$$\underset{\sim}{F}_p = -\nabla V_p$$

$$\underset{\sim}{V}_p = \frac{1}{2} m \langle \underset{\sim}{u}_f \cdot \underset{\sim}{u}_f \rangle$$

$$\underset{\sim}{u}_f = \frac{1}{2}\left(\hat{\underset{\sim}{u}}_f e^{-i\omega_f t} + \hat{\underset{\sim}{u}}_f^* e^{i\omega_f t}\right) = \mathrm{Re}\{\hat{\underset{\sim}{u}}_f e^{-i\omega_f t}\}$$

$$V_p = \frac{m}{4} |\hat{\underset{\sim}{u}}_f|^2$$

$$\hat{\underset{\sim}{u}}_f = \frac{q}{m(-i\omega_f)} \cdot \hat{\underset{\sim}{E}}_f \qquad \text{FROM EQ. of motion}$$

$$\boxed{\bar{V}_p = \frac{q^2 |\hat{\underset{\sim}{E}}_f|^2}{4\omega_f^2 m}} = \frac{1}{4}\left|\frac{q\hat{\underset{\sim}{A}}}{mc^2}\right|^2$$

* PONDEROMOTIVE FORCE same sign for e, i
* much bigger for electrons
* pushes particles out of high field region.