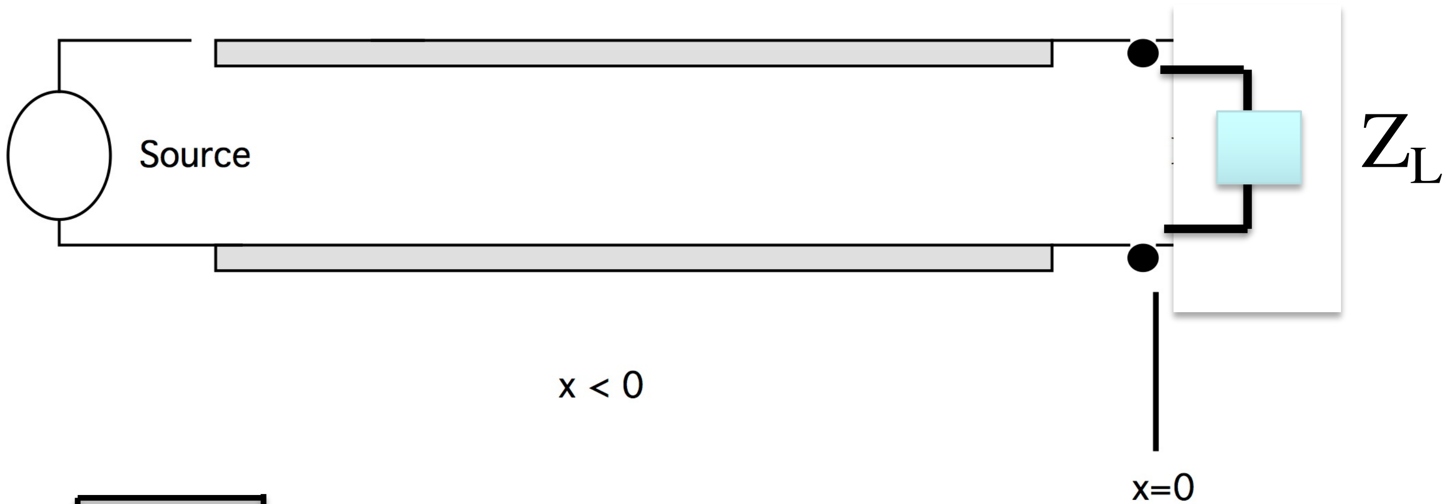
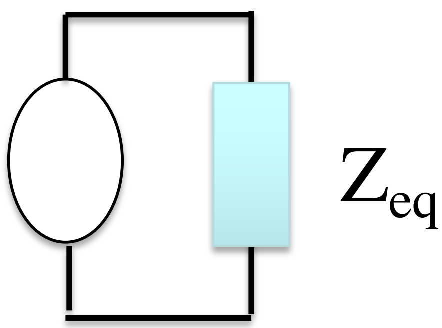
What about AC signals?





The impedance presented to the source is modified to Z_{eq}, Depends on the load, the length and the characteristic impedance of the line.

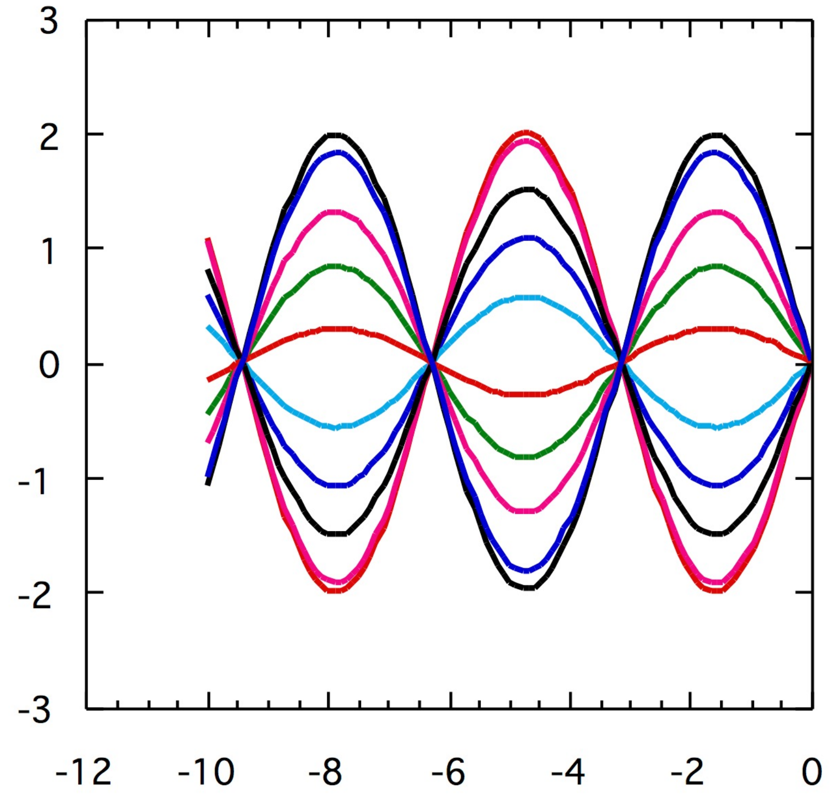
Standing Waves

Plots of Ex(z,t) at different time

$$V = \mathrm{Re}\left\{ \hat{V}_{inc} \left(e^{-jkz} + \rho e^{jkz} \right) e^{j\omega t} \right\}$$

$$I = \mathrm{Re}\left\{ \frac{\hat{V}_{inc}}{Z_0} \left(e^{-jkz} - \rho e^{jkz} \right) e^{j\omega t} \right\}$$

V(z,t)



$$\rho = -1$$

Nodes are short circuits

Anti nodes are open circuits

Z [m]

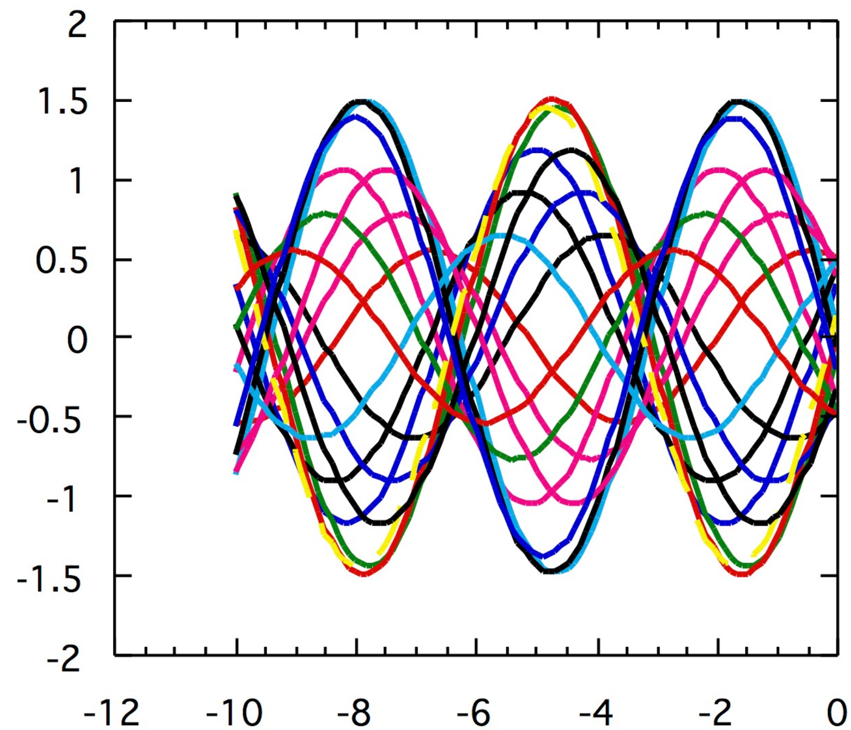
Standing Waves

Plots of Ex(z,t) at different time

$$V = \mathrm{Re}\left\{ \hat{V}_{inc} \left(e^{-jkz} + \rho e^{jkz} \right) e^{j\omega t} \right\}$$

$$I = \mathrm{Re}\left\{ \frac{\hat{V}_{inc}}{Z_0} \left(e^{-jkz} - \rho e^{jkz} \right) e^{j\omega t} \right\}$$

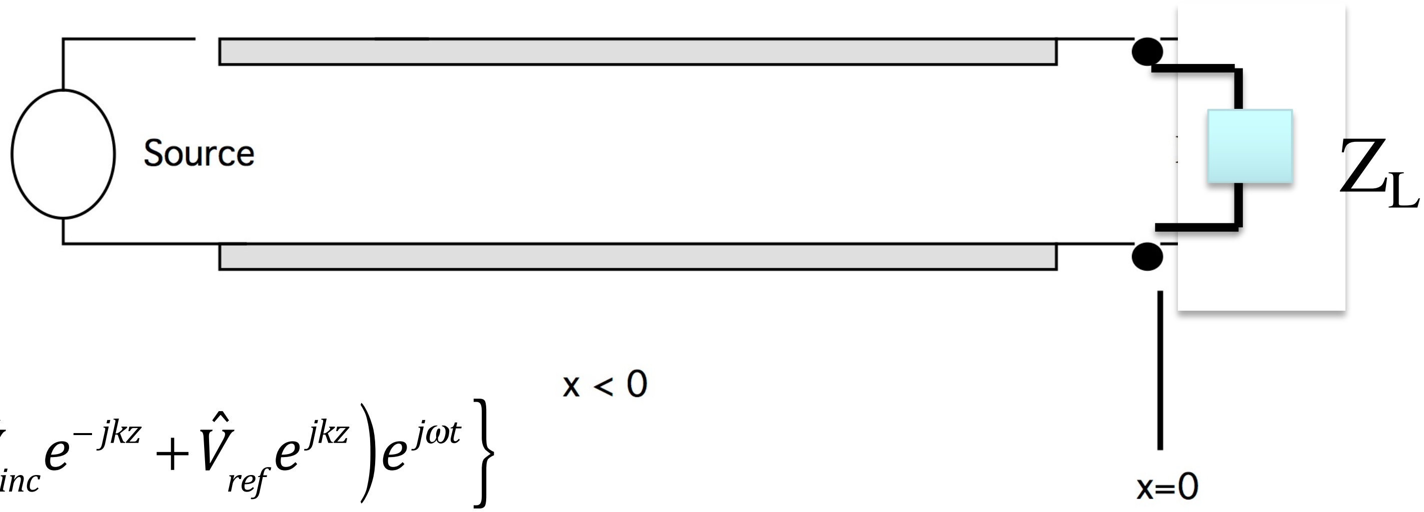
V(z,t)



$$\rho = -0.5$$

Impedance is complex and depends on where V/I is measured

Z [m]



$$V = \mathrm{Re}\left\{\left(\hat{V}_{inc}e^{-jkz} + \hat{V}_{ref}e^{jkz}\right)e^{j\omega t}\right\}$$

$$V = \mathrm{Re}\left\{\hat{V}_{inc}\left(e^{-jkz} + \rho e^{jkz}\right)e^{j\omega t}\right\} = \mathrm{Re}\left\{\hat{V}(z)e^{j\omega t}\right\}$$

$$I = \mathrm{Re}\left\{\frac{\hat{V}_{inc}}{Z_0}\left(e^{-jkz} - \rho e^{jkz}\right)e^{j\omega t}\right\} = \mathrm{Re}\left\{\hat{I}(z)e^{j\omega t}\right\}$$

The reflection coefficient at th

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

At z = -L

$$Z_{eq} = \frac{\hat{V}(-L)}{\hat{I}(-L)} = Z_0 \left.\frac{\left(e^{-jkz} + \rho e^{jkz}\right)}{\left(e^{-jkz} - \rho e^{jkz}\right)}\right|_{z=-L} = Z_0 \frac{1 + \rho e^{-2jkL}}{1 - \rho e^{-2jkL}}$$

The reflection coefficient

$$\rho(L) = \frac{Z_{eq} - Z_0}{Z_{eq} + Z_0} = \rho e^{-2jkL}$$

Equivalent Impedance

$$Z_{eq} = Z_0 \frac{1+\rho e^{-2jkL}}{1-\rho e^{-2jkL}}$$

$$Z_{eq} = Z_0 \frac{\left(Z_L + Z_0\right)e^{jkL} + \left(Z_L - Z_0\right)e^{-jkL}}{\left(Z_L + Z_0\right)e^{jkL} - \left(Z_L - Z_0\right)e^{-jkL}}$$

$$Z_{eq} = Z_0 \frac{Z_L \cos\left(kL\right) + jZ_0 \sin\left(kL\right)}{jZ_L \sin\left(kL\right) + Z_0 \cos\left(kL\right)}$$

If line has losses effect of reflections is diminished

$$k = k' - jk'', \quad e^{-2jkL} = e^{-2k''L} e^{-2jk'L}$$

$$Z_{eq} = Z_0 \frac{1+\left(\rho e^{-2k''L}\right)e^{-2jk'L}}{1-\left(\rho e^{-2k''L}\right)e^{-2jk'L}}$$

If L $\rightarrow$ 0, $\sin(kL) \rightarrow 0$, $\cos(kL) \rightarrow 1$, $\quad Z_{eq} \rightarrow Z_L$

If kL $\rightarrow \frac{\pi}{2}$, $\sin(kL) \rightarrow 1$, $\cos(kL) \rightarrow 0$ $\quad Z_{eq} \rightarrow Z_0^2 / Z_L$

Special Cases

$$Z_{eq} = Z_0 \frac{1+\rho e^{-2jk_1L}}{1-\rho e^{-2jk_1L}}$$

$$L = n\lambda/2, \quad 2k_1L = 2\pi n \qquad Z_{eq} = Z_0 \frac{1+\rho}{1-\rho} = Z_L$$

Remember half-wave wind

$$L = \left(n+\frac{1}{2}\right)\lambda/2, \quad 2k_1L = 2\pi\left(n+\frac{1}{2}\right) \qquad Z_{eq} = Z_0 \frac{1-\rho}{1+\rho} = Z_0^2/Z_L$$

Quarter-wave transformer

Admittance vs Impedance

$$I = \mathrm{Re}\left\{\left(\hat{I}_{inc}e^{-jkz} + \hat{I}_{ref}e^{jkz}\right)e^{j\omega t}\right\}$$

$$V = \mathrm{Re}\left\{Z_0\left(\hat{I}_{inc}e^{-jkz} - \hat{I}_{inc}e^{jkz}\right)e^{j\omega t}\right\}$$

Same formulas apply to current amplitudeS.

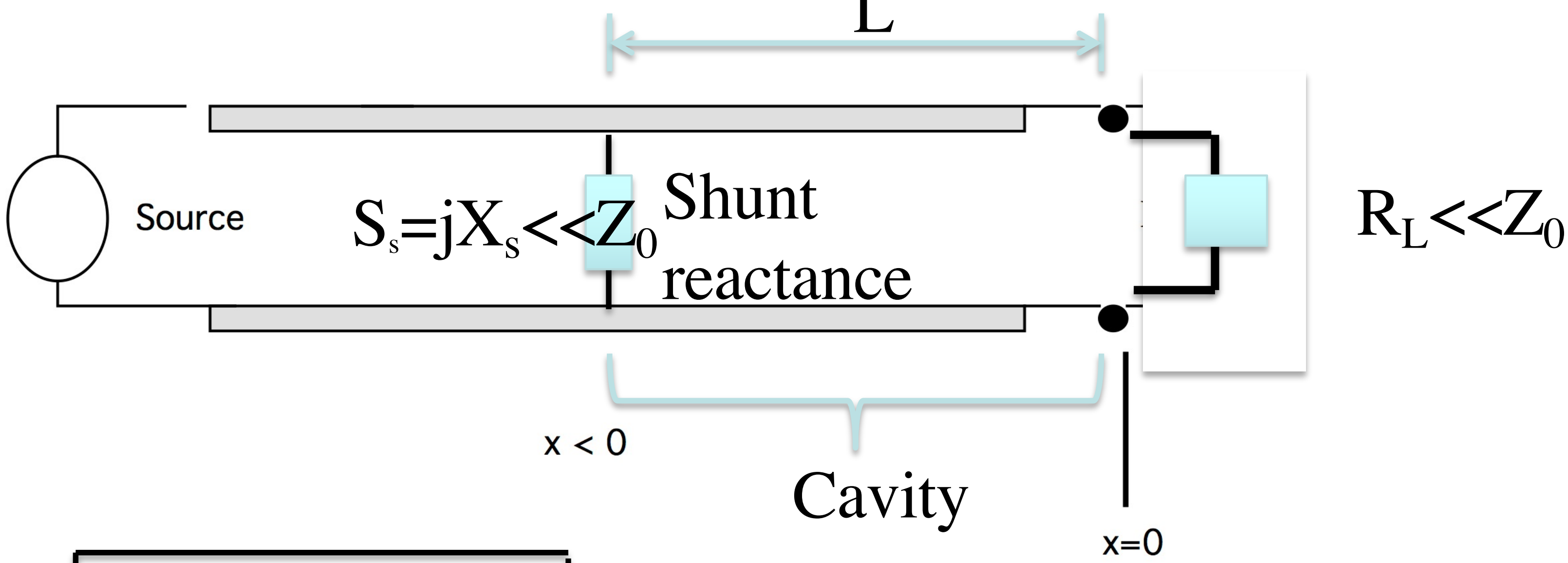
$$I = \mathrm{Re}\left\{\hat{I}_{inc}\left(e^{-jkz} + \rho_I e^{jkz}\right)e^{j\omega t}\right\} = \mathrm{Re}\left\{\hat{I}(z)e^{j\omega t}\right\}$$

$$V = \mathrm{Re}\left\{Z_0\hat{I}_{inc}\left(e^{-jkz} - \rho_I e^{jkz}\right)e^{j\omega t}\right\} = \mathrm{Re}\left\{\hat{V}(z)e^{j\omega t}\right\}$$

Current reflection coefficient = - voltage reflection coefficient

$$\rho_I = -\rho \quad \rho_I = -\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_L - Y_0}{Y_L + Y_0}$$

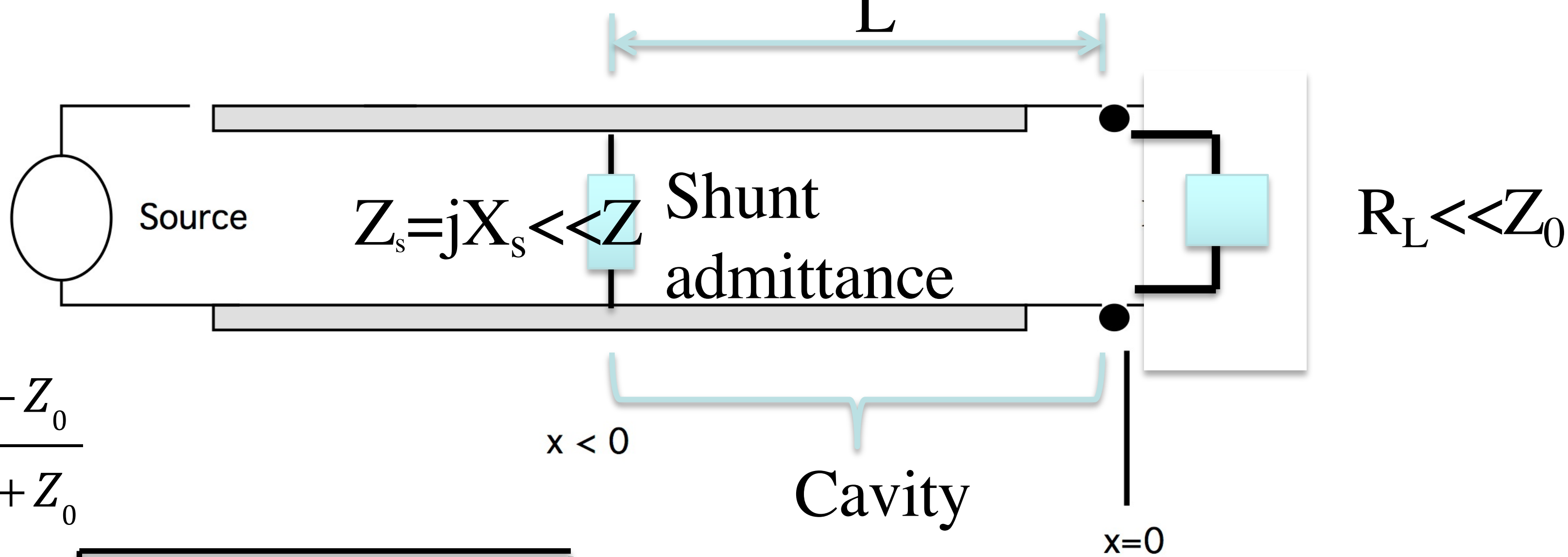
High Q cavity model



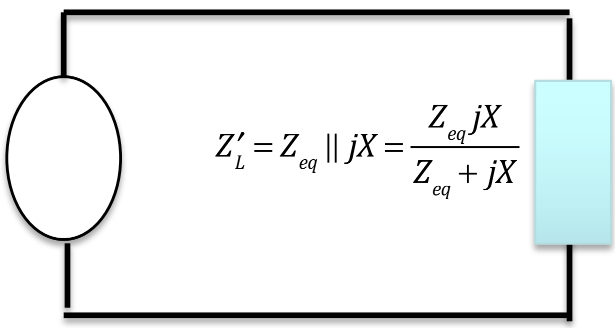
$$Z'_L = Z_{eq} \,||\, jX = \frac{Z_{eq} jX}{Z_{eq} + jX}$$

$$\rho == \frac{Z'_{eq} - Z_0}{Z'_{eq} + Z_0}$$

High Q Cavity Model



$$\rho_{cav} = \frac{\left(Z_{eq} \,||\, jX_s\right) - Z_0}{\left(Z_{eq} \,||\, jX_s\right)_s + Z_0}$$



$$Z_{eq} = Z_0 \frac{R_L \cos(kL) + jZ_0 \sin(kL)}{jR_L \sin(kL) + Z_0 \cos(kL)} \simeq jZ_0 \tan(kL) + R_L$$

$$\rho_{cav} \simeq -\frac{j\left(Z_0 \tan(kL) + X_s\right) + R_L - X_s^2 / Z_0}{j\left(Z_0 \tan(kL) + X_s\right) + R_L + X_s^2 / Z_0} = -\frac{j2\left(\omega - \omega_c\right)/\omega_c + Q_{\text{int}}^{-1} - Q_{ext}^{-1}}{j2\left(\omega - \omega_c\right)/\omega_c + Q_{\text{int}}^{-1} + Q_{ext}^{-1}}$$

Smith Chart

Graphically solves the following bi-linear formulas

$$\frac{Z_{eq}(l)}{Z_0} = \frac{1+\left(\rho\, e^{-2jkl}\right)}{1-\left(\rho\, e^{-2jkl}\right)}$$

$$\rho = \frac{\left(Z_L / Z_0\right)-1}{\left(Z_L / Z_0\right)+1}$$

Note: works for admittance too.

$$\frac{Y_{eq}(l)}{Y_0} = \left(\frac{Z_{eq}(l)}{Z_0}\right)^{-1} = \frac{1-\left(\rho\, e^{-2jkl}\right)}{1+\left(\rho\, e^{-2jkl}\right)}$$

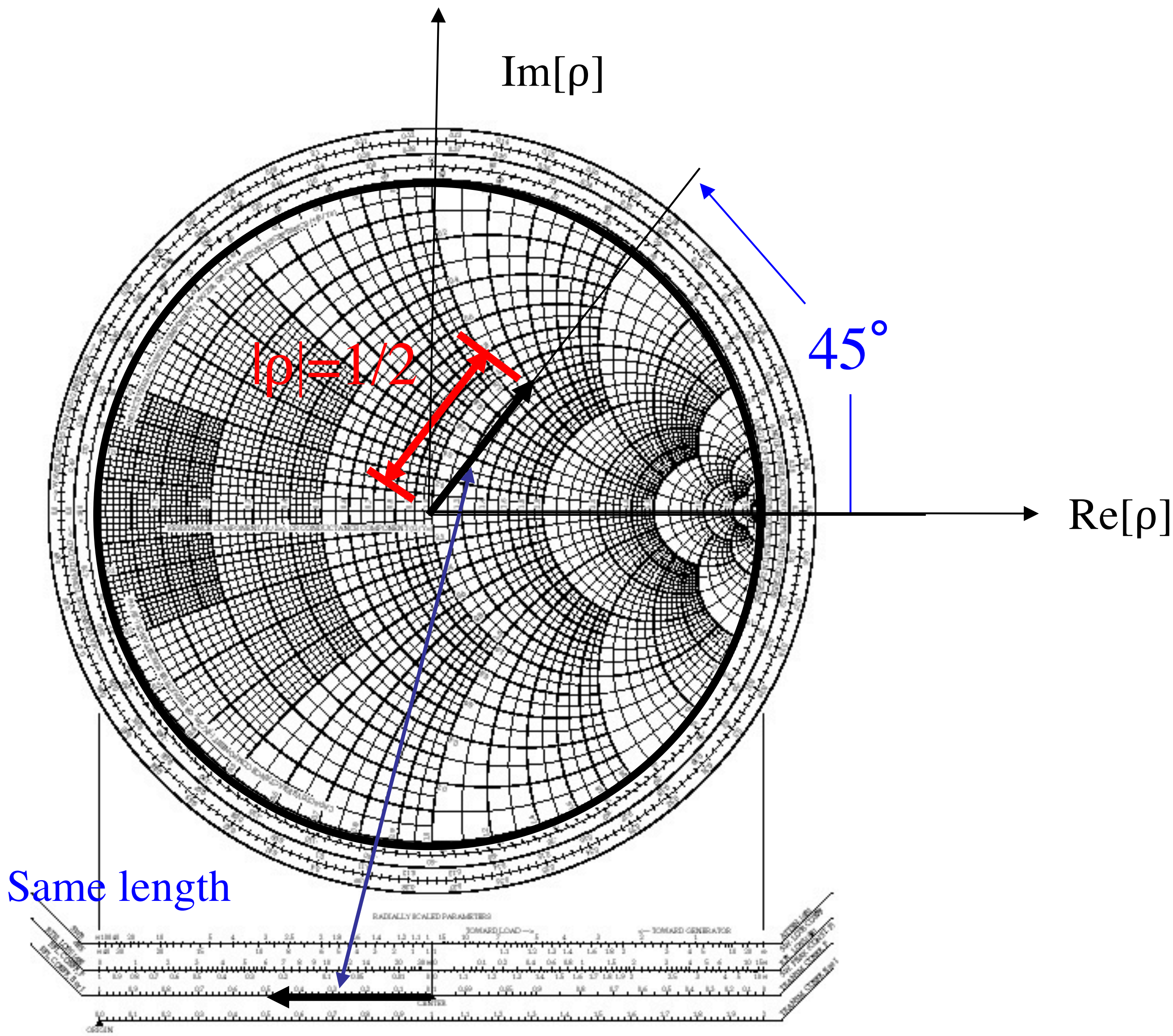
Just switch sign of ρ

$$\rho \quad \rightarrow \quad -\rho$$

Smith chart is the interior of the unit circle in the complex plane

Example:

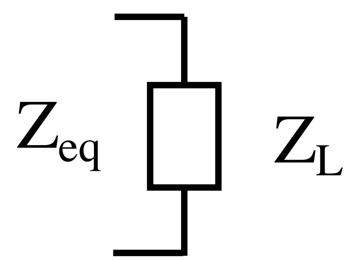
$$\rho = \frac{1}{2} e^{j\frac{\pi}{4}}$$



Find Z_L given ρ

$$\frac{Z_{eq}(l)}{Z_0} = \frac{1+\left(\rho\, e^{-2jkl}\right)}{1-\left(\rho\, e^{-2jkl}\right)}$$

Note: $Z_{eq}(l=0) = Z_L$

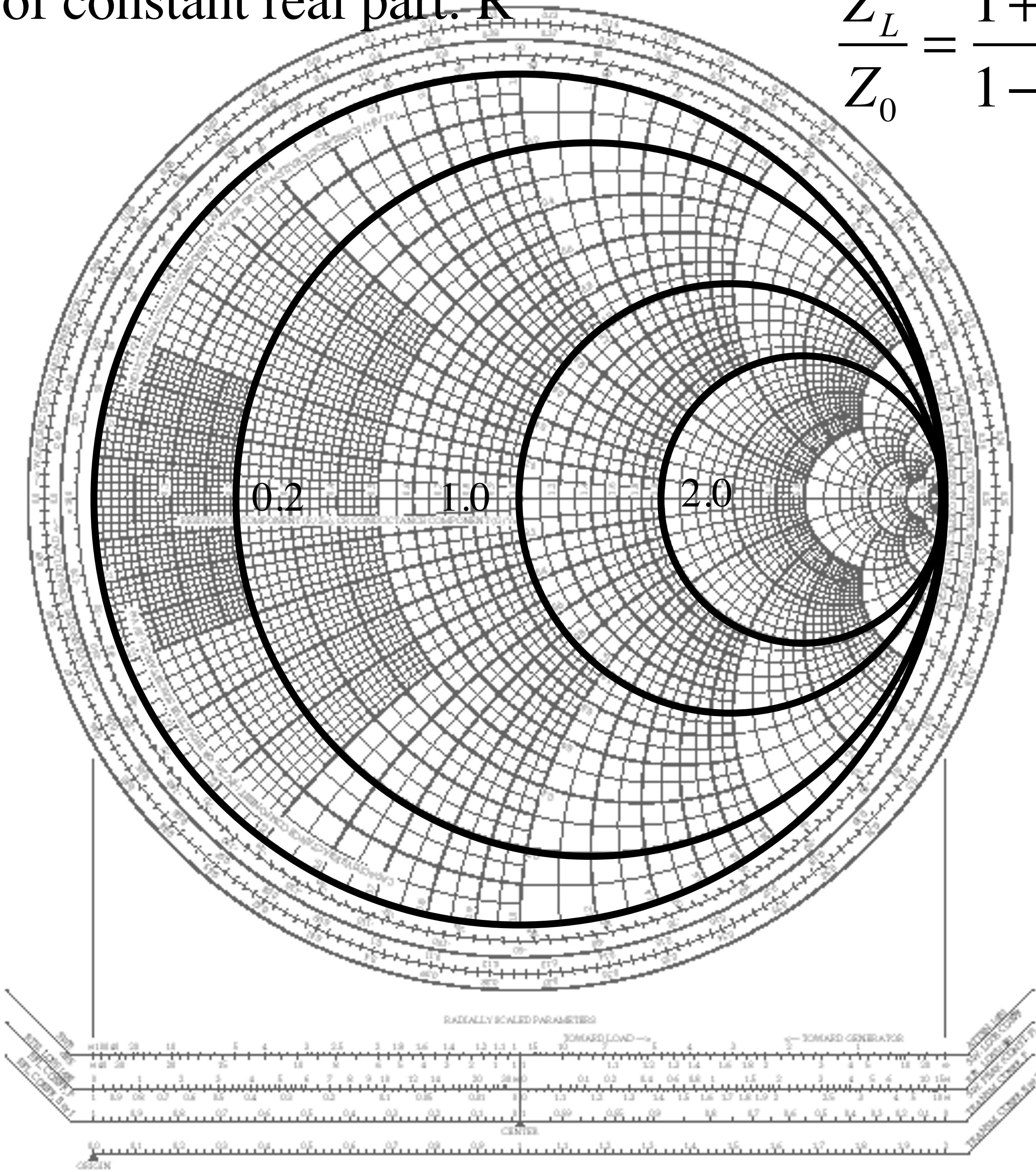


Find real and Imaginary parts:

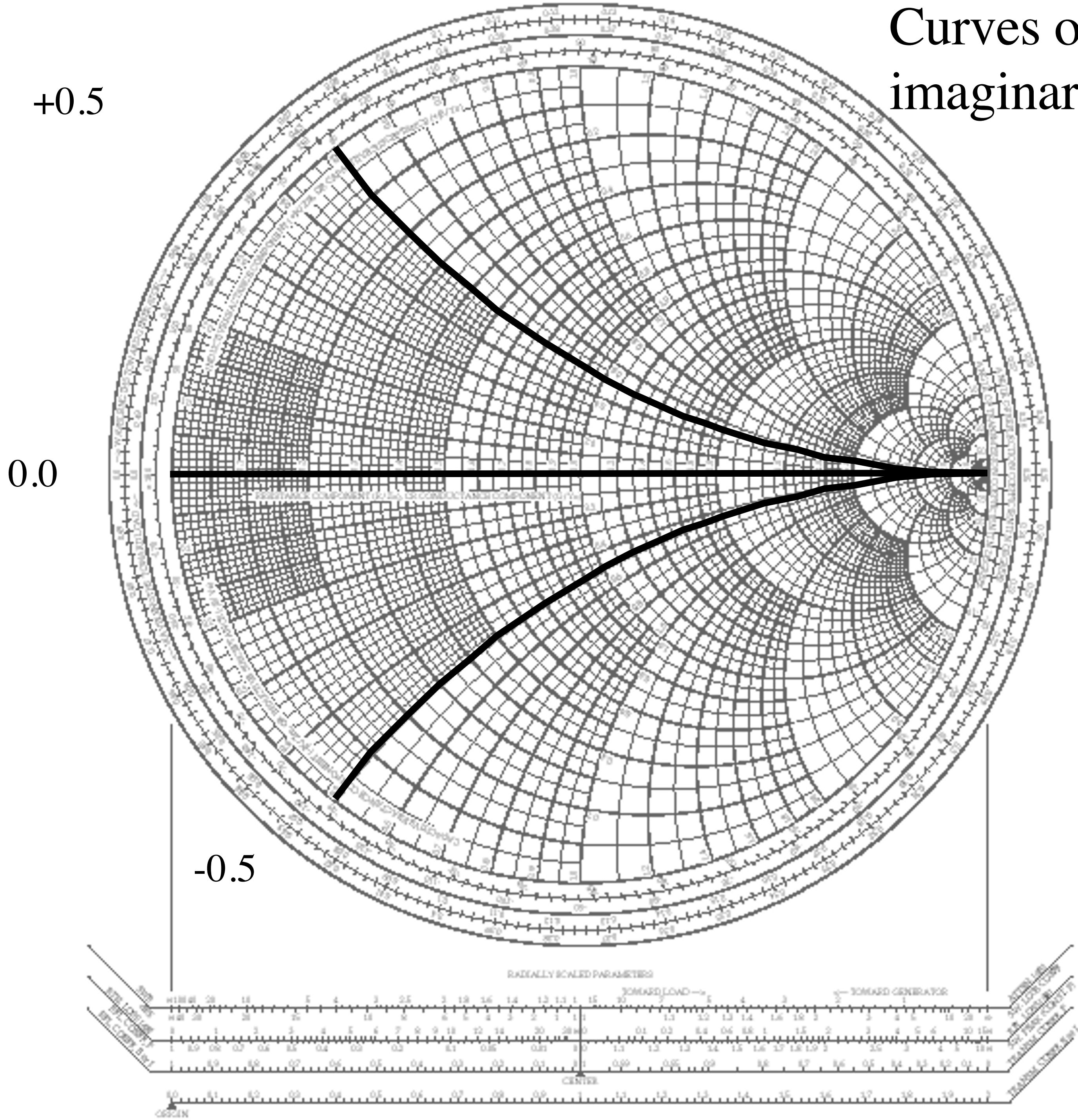
$$\frac{Z_L}{Z_0} = \frac{1+\rho}{1-\rho} = R + jX$$

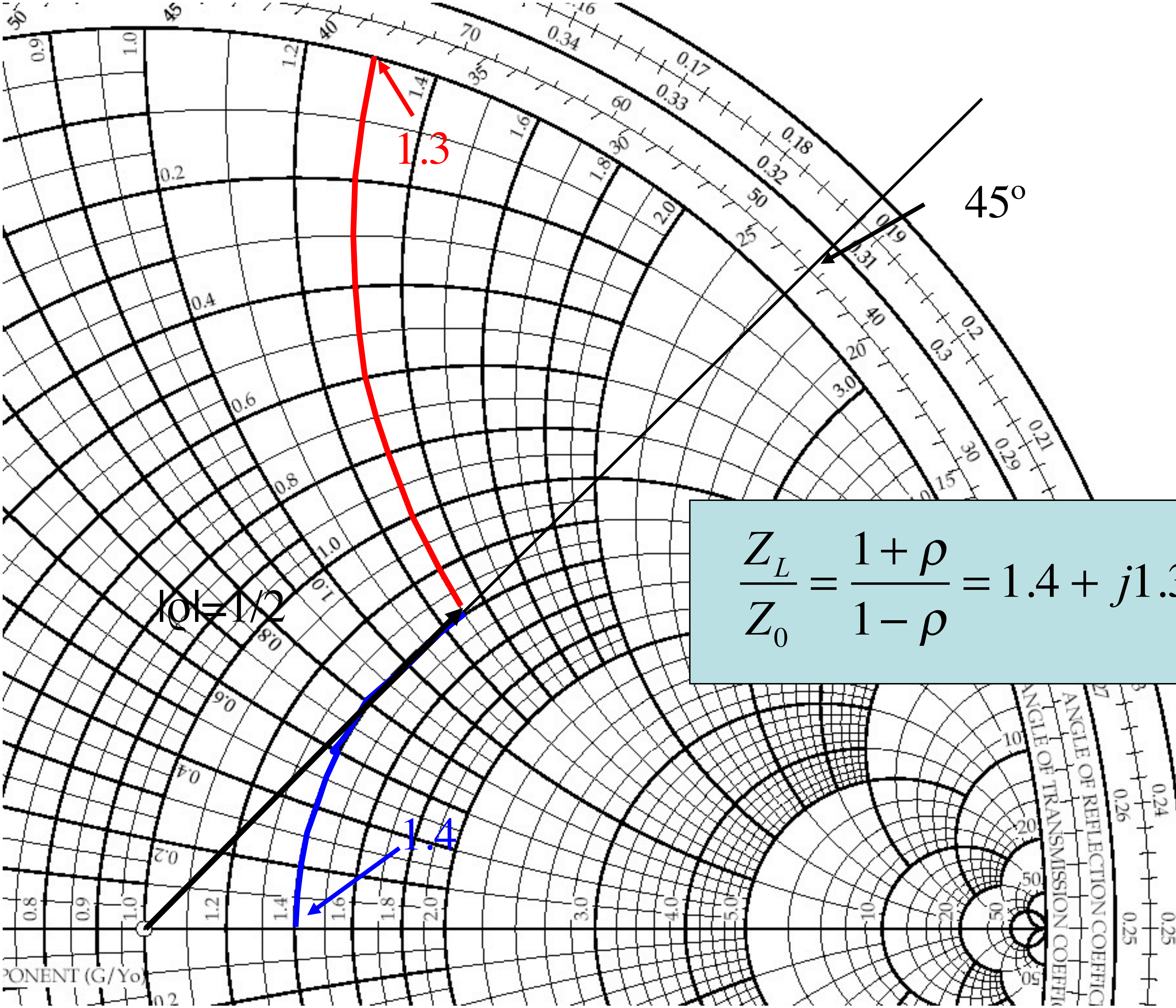
Curves of constant real part: R

$$\frac{Z_L}{Z_0} = \frac{1+\rho}{1-\rho} = R + jX$$



Curves of constant imaginary part: X



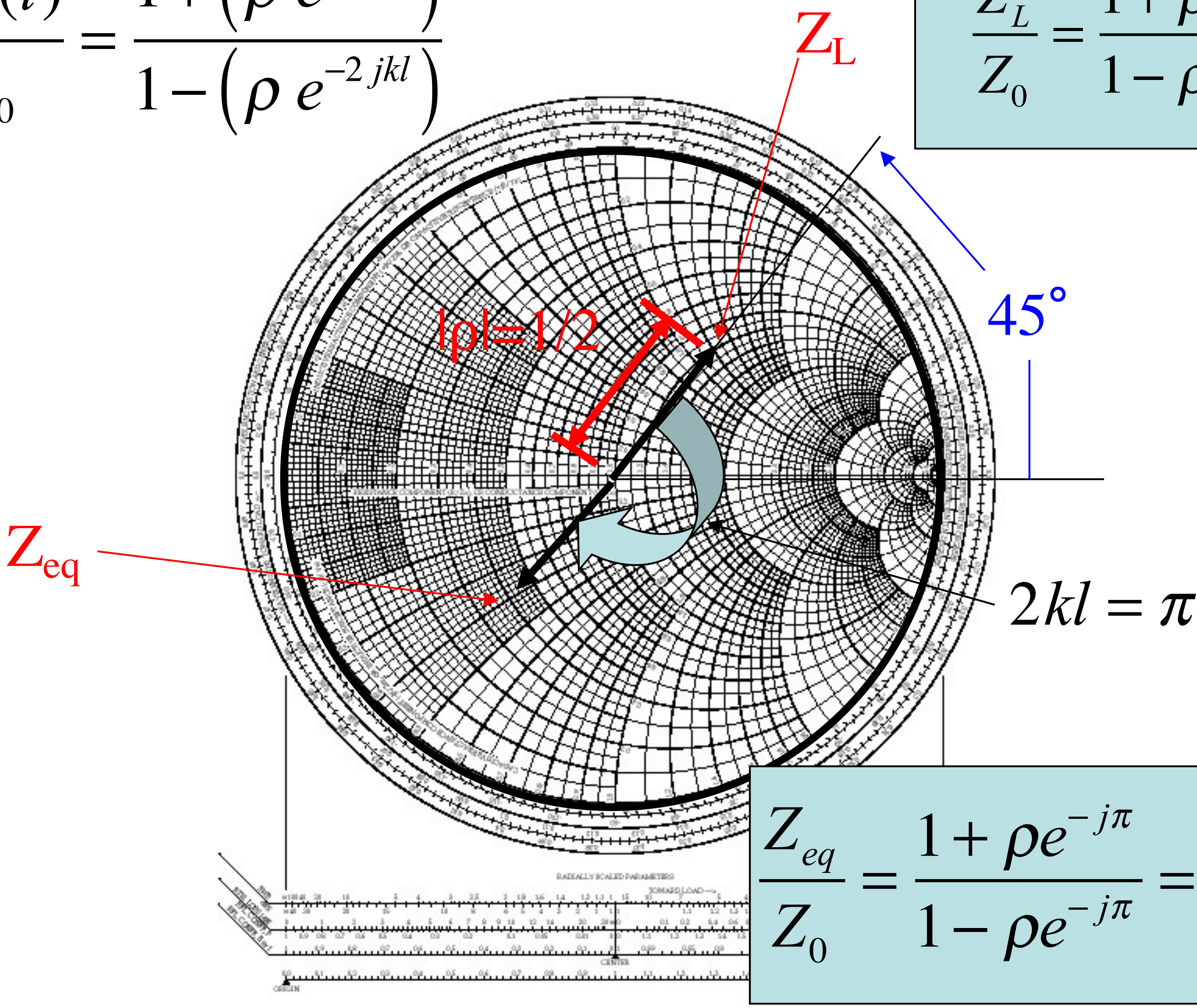


$$\frac{Z_L}{Z_0} = \frac{1+\rho}{1-\rho} = 1.4 + j1.3$$

What is Z_{eq} at $l = \lambda/4$ from the load?

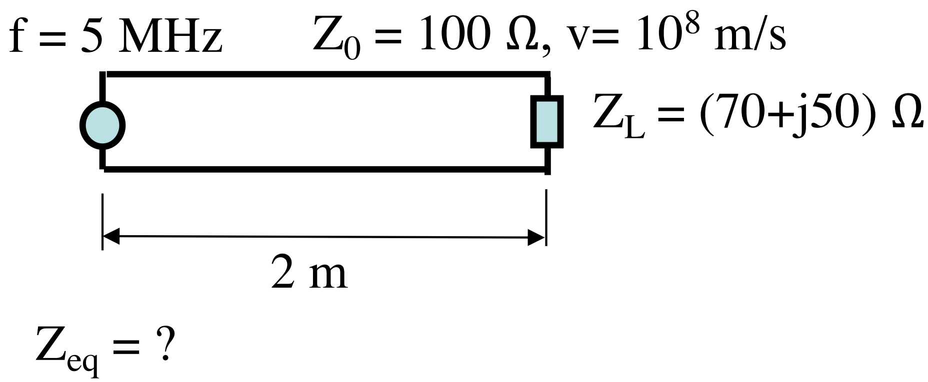
$$\frac{Z_{eq}(l)}{Z_0} = \frac{1+\left(\rho\, e^{-2jkl}\right)}{1-\left(\rho\, e^{-2jkl}\right)}$$

$$\frac{Z_L}{Z_0} = \frac{1+\rho}{1-\rho} = 1.4 + j1.3$$



$$\frac{Z_{eq}}{Z_0} = \frac{1+\rho e^{-j\pi}}{1-\rho e^{-j\pi}} = .38 - j.34$$

Sample Problem: find Z_{eq}



Method 1:

$$Z_{eq}(l) = Z_o \frac{Z_L \cos kl + jZ_0 \sin kl}{Z_0 \cos kl + jZ_L \sin kl}$$

$$k = 2\pi / \lambda$$

$$\lambda = v / f = 10^8 / 5 \times 10^6 = 20m$$

$$kl = 0.628$$

$$or$$

$$l / \lambda = .1$$

$$\cos kl = 0.81$$

$$\sin kl = 0.59$$

$$Z_{eq}(2) = Z_o \frac{56.63 + j99.5}{51.50 + j41.14}$$

$$Z_{eq}(2) = (161 + j64)\Omega$$

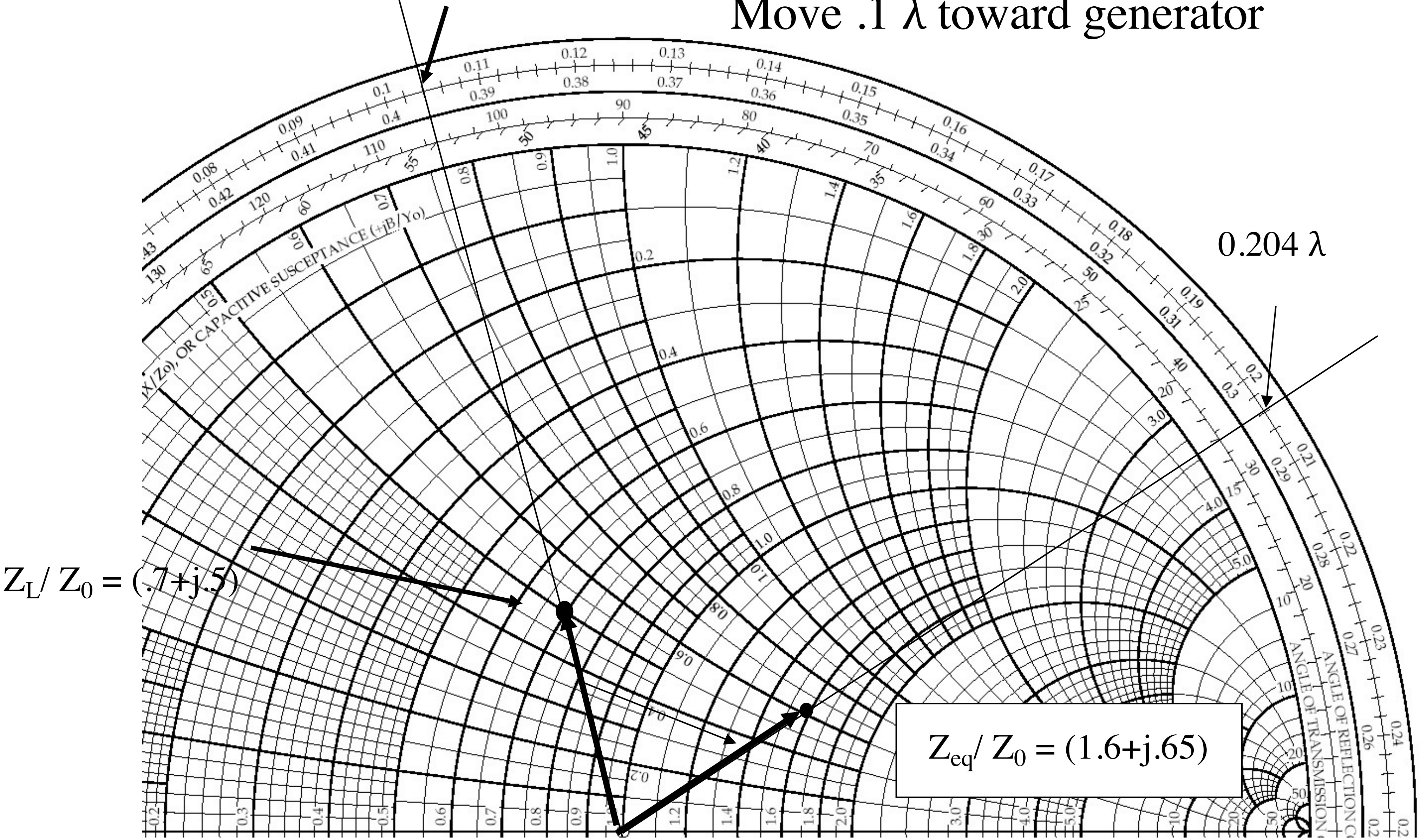
Method 2: Smith Chart

$Z_L / Z_0 = (70+j50)/100$

$Z_L / Z_0 = (.7+j.5)$

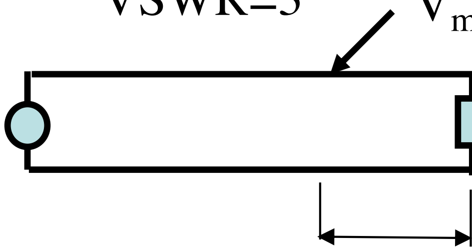
Move .1 λ toward generator

0.104 λ



Standing Wave Problem

VSWR=3

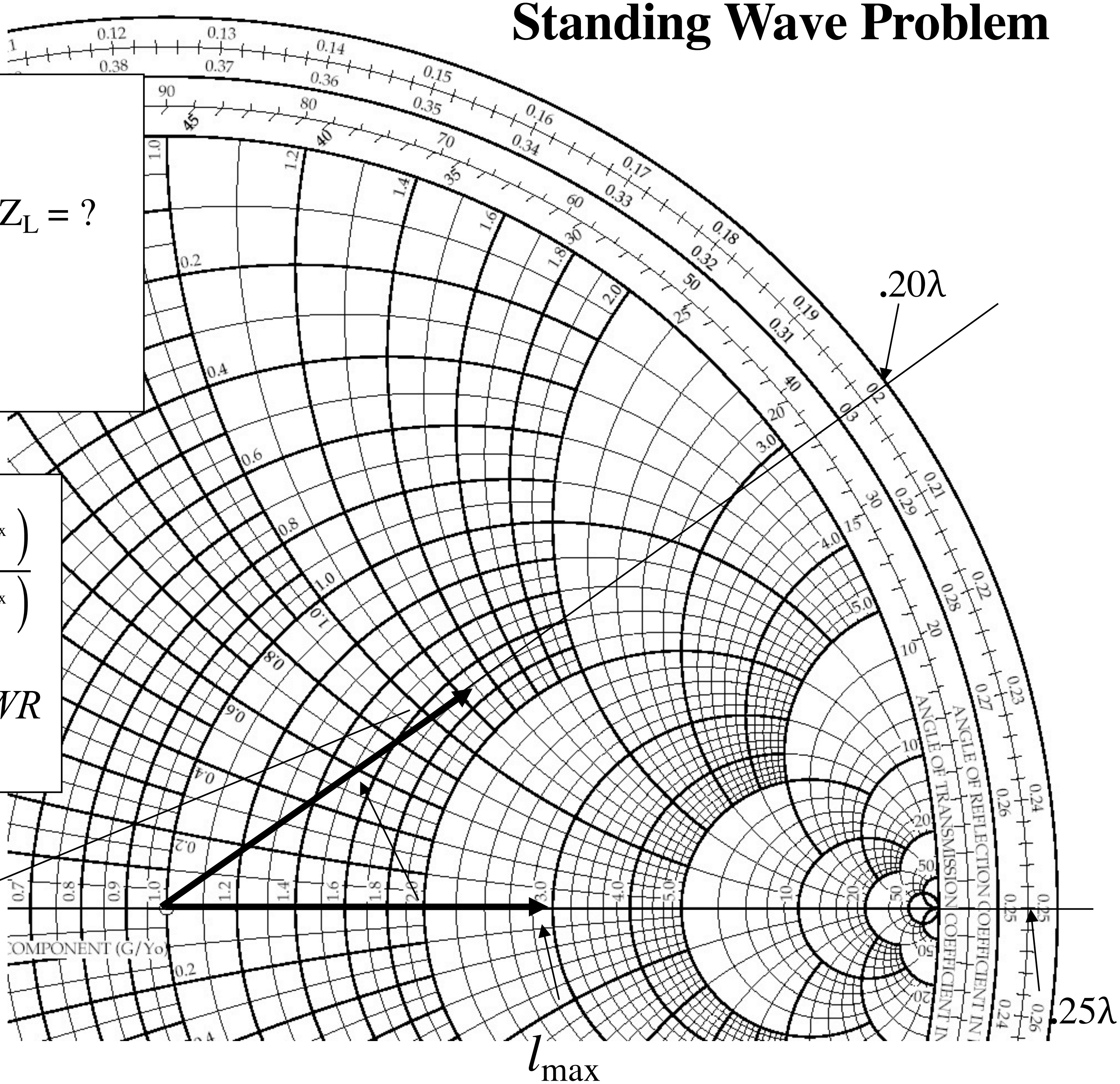


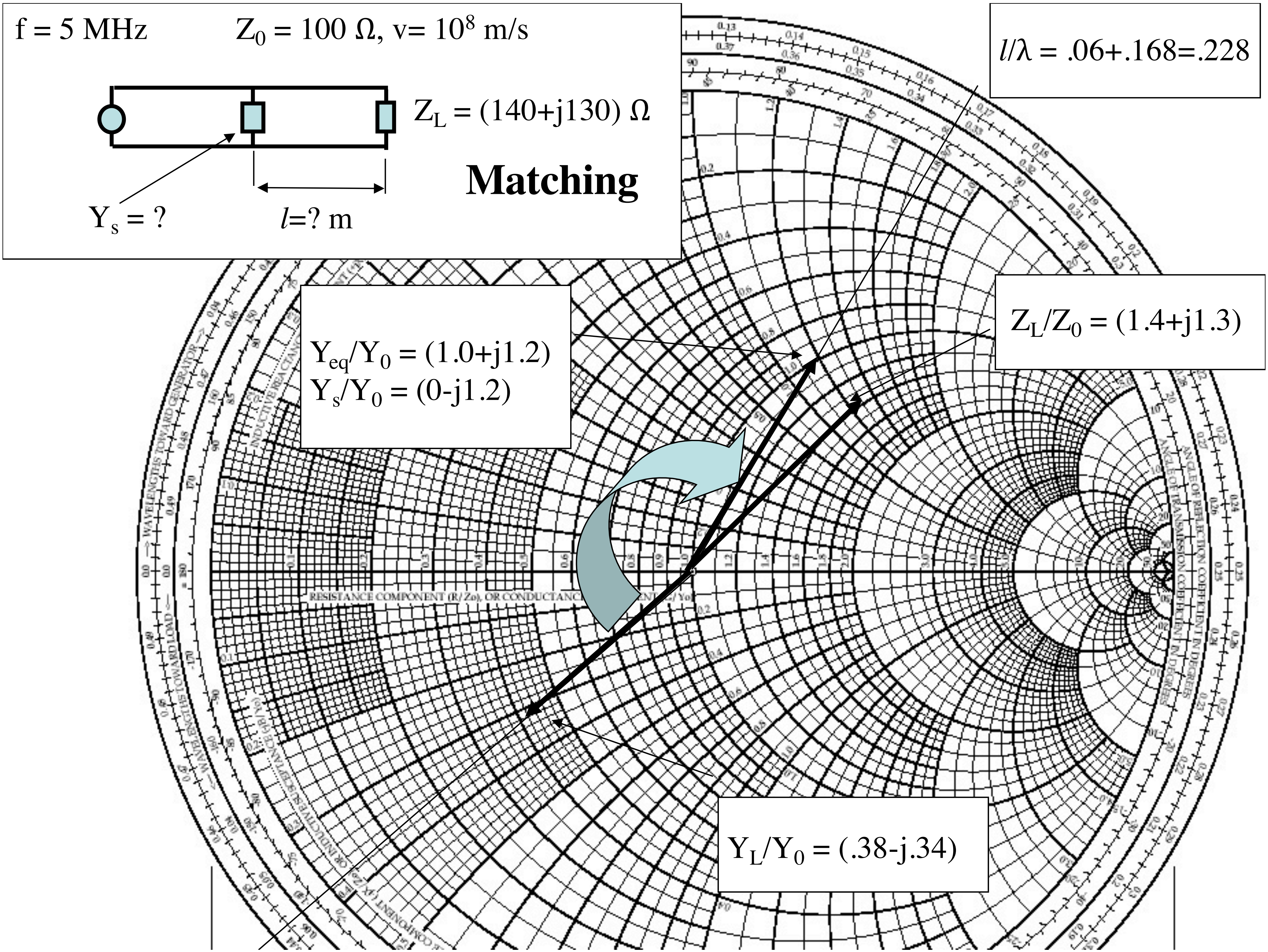
λ= 20m,
Z_0=100 Ω

$$\frac{Z_{eq}(l_{\max})}{Z_0} = \frac{1+\left(\rho\, e^{-2jkl_{\max}}\right)}{1-\left(\rho\, e^{-2jkl_{\max}}\right)}$$

$$\frac{Z_{eq}(l_{\max})}{Z_0} = \frac{1+|\rho|}{1-|\rho|} = VSWR$$

1.7+j1.3
Z_L=170+j130 Ω





f = 5 MHz
Z_0 = 100 Ω, v= 10^8 m/s
Z_L = (140+j130) Ω
Matching
Y_s = ?
l=? m
l/λ = .06+.168=.228
Z_L/Z_0 = (1.4+j1.3)
Y_{eq}/Y_0 = (1.0+j1.2)
Y_s/Y_0 = (0-j1.2)
Y_L/Y_0 = (.38-j.34)

Shunt admittance