Smith Chart

Graphically solves the following bi-linear formulas

$$\frac{Z_{eq}(l)}{Z_0} = \frac{1+\left(\rho\, e^{-2jkl}\right)}{1-\left(\rho\, e^{-2jkl}\right)}$$

$$\rho = \frac{\left(Z_L / Z_0\right)-1}{\left(Z_L / Z_0\right)+1}$$

Note: works for admittance too.

$$\frac{Y_{eq}(l)}{Y_0} = \left(\frac{Z_{eq}(l)}{Z_0}\right)^{-1} = \frac{1-\left(\rho\, e^{-2jkl}\right)}{1+\left(\rho\, e^{-2jkl}\right)}$$

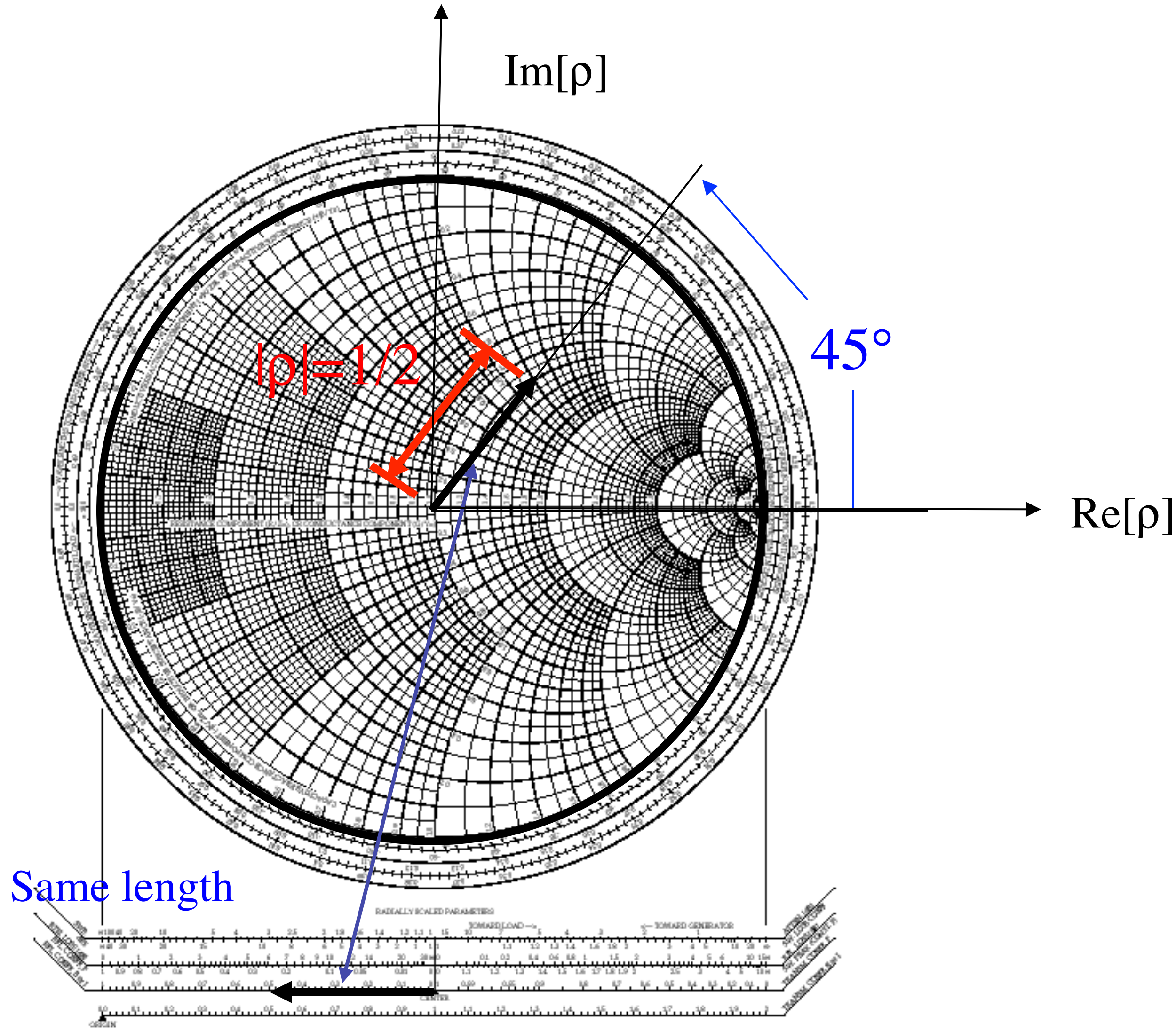
Just switch sign of ρ

$$\rho \quad \rightarrow \quad -\rho$$

Smith chart is the interior of the unit circle in the complex plane

Example:

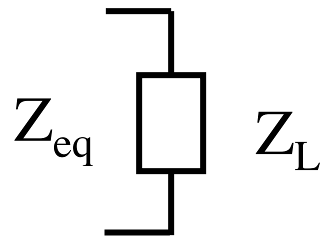
$$\rho = \frac{1}{2} e^{j\frac{\pi}{4}}$$



Find Z_L given ρ

$$\frac{Z_{eq}(l)}{Z_0} = \frac{1+\left(\rho\, e^{-2jkl}\right)}{1-\left(\rho\, e^{-2jkl}\right)}$$

Note: $Z_{eq}(l=0) = Z_L$

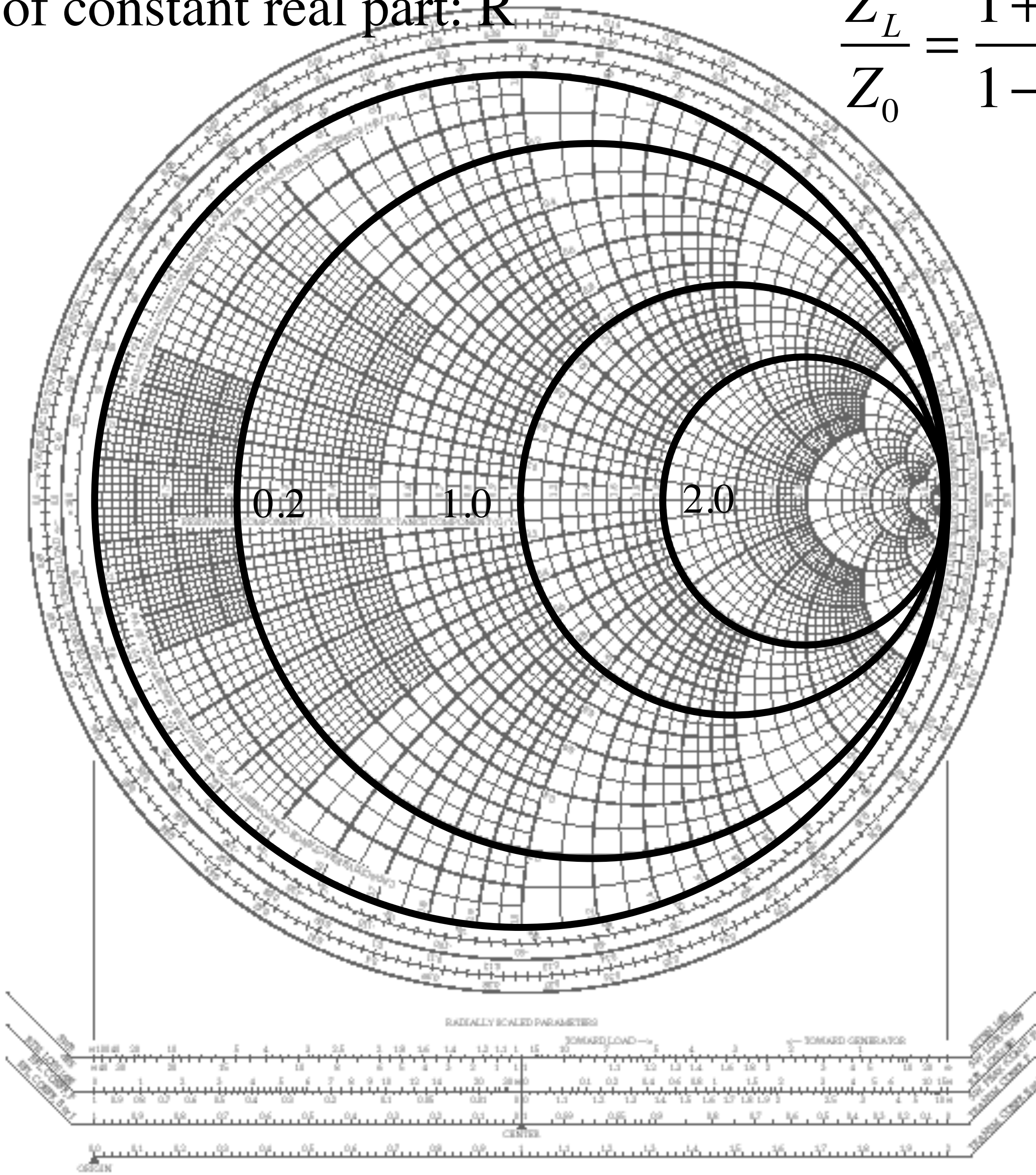


Find real and Imaginary parts:

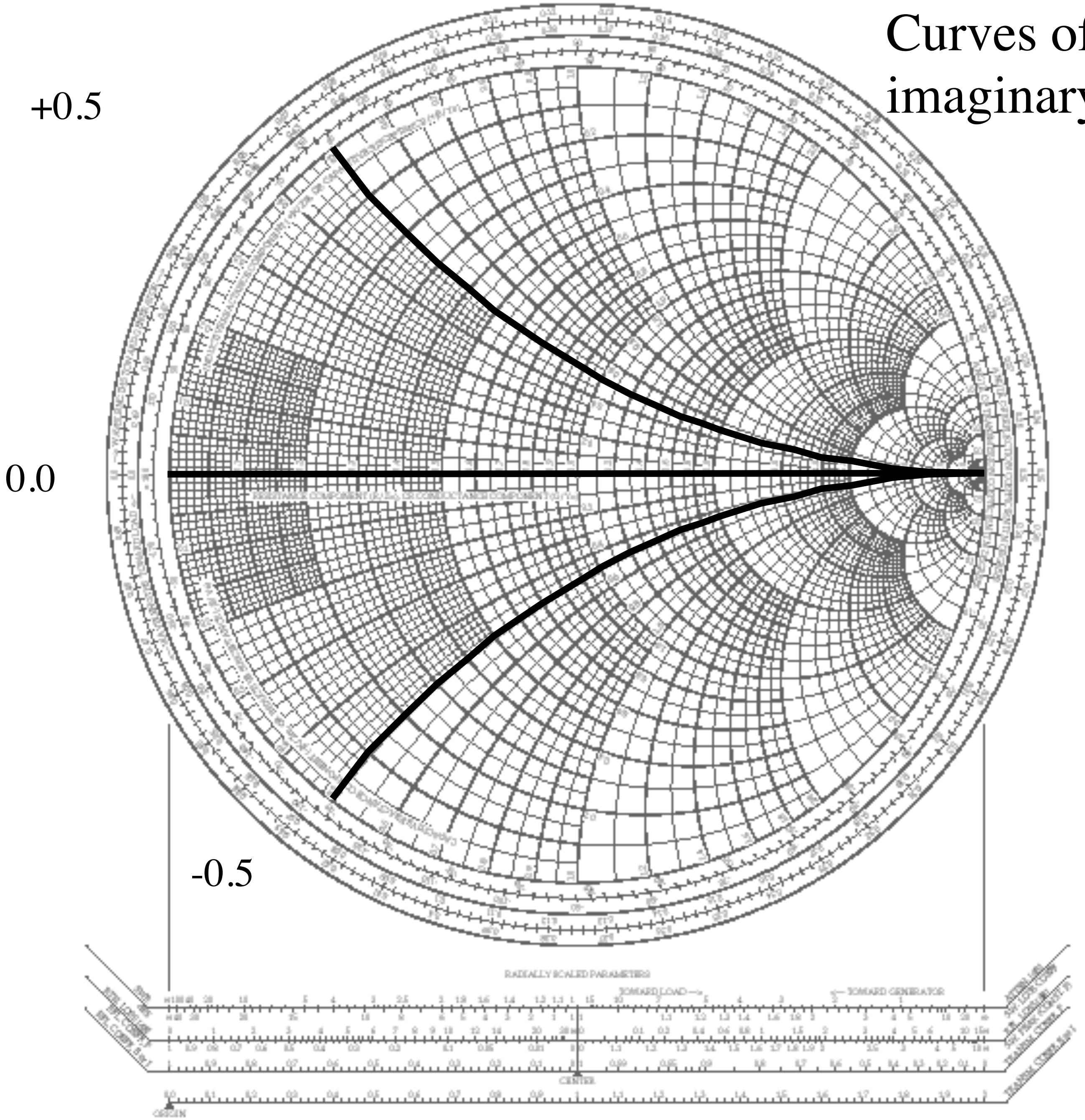
$$\frac{Z_L}{Z_0} = \frac{1+\rho}{1-\rho} = R + jX$$

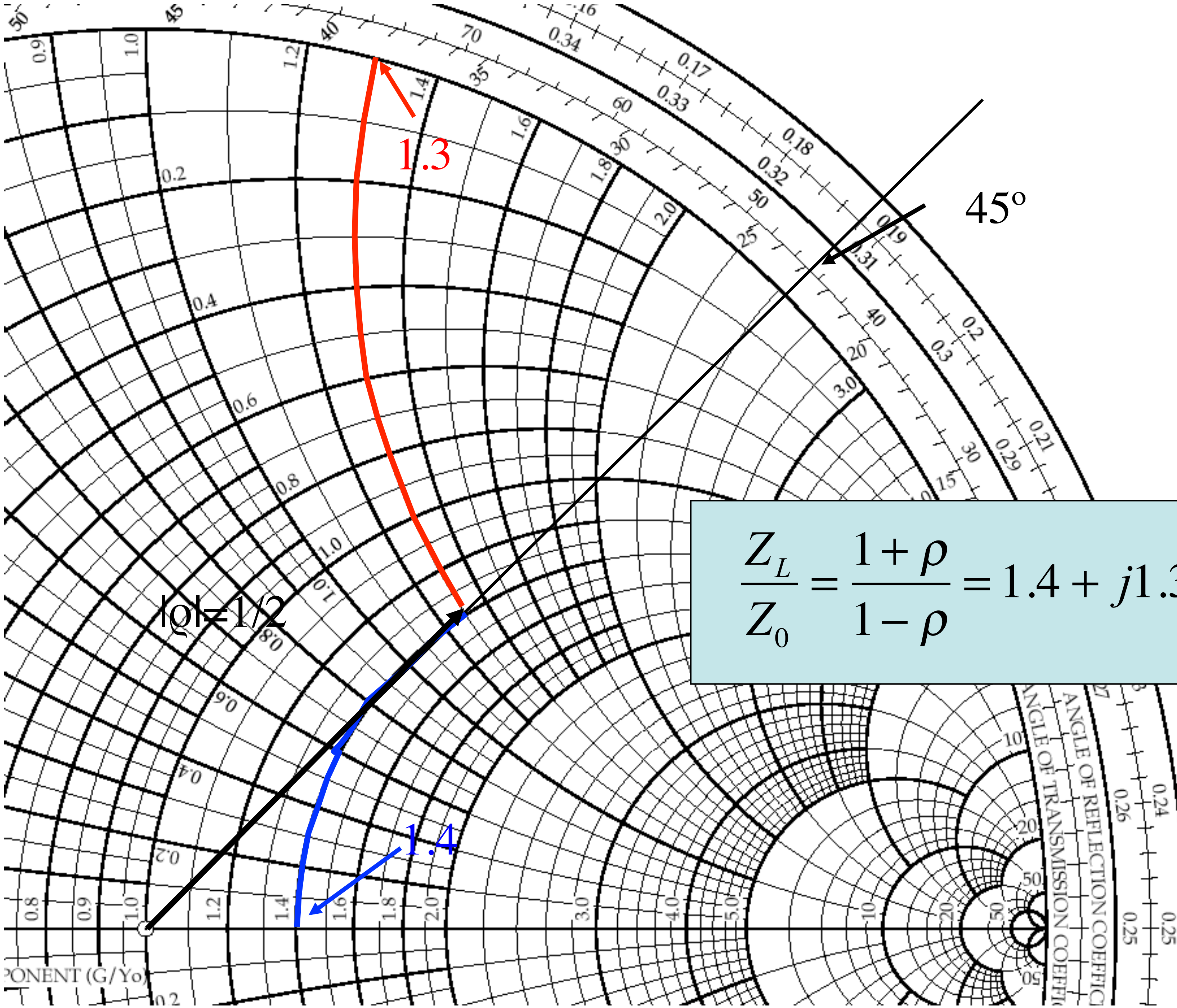
Curves of constant real part: R

$$\frac{Z_L}{Z_0} = \frac{1+\rho}{1-\rho} = R + jX$$



Curves of constant imaginary part: X



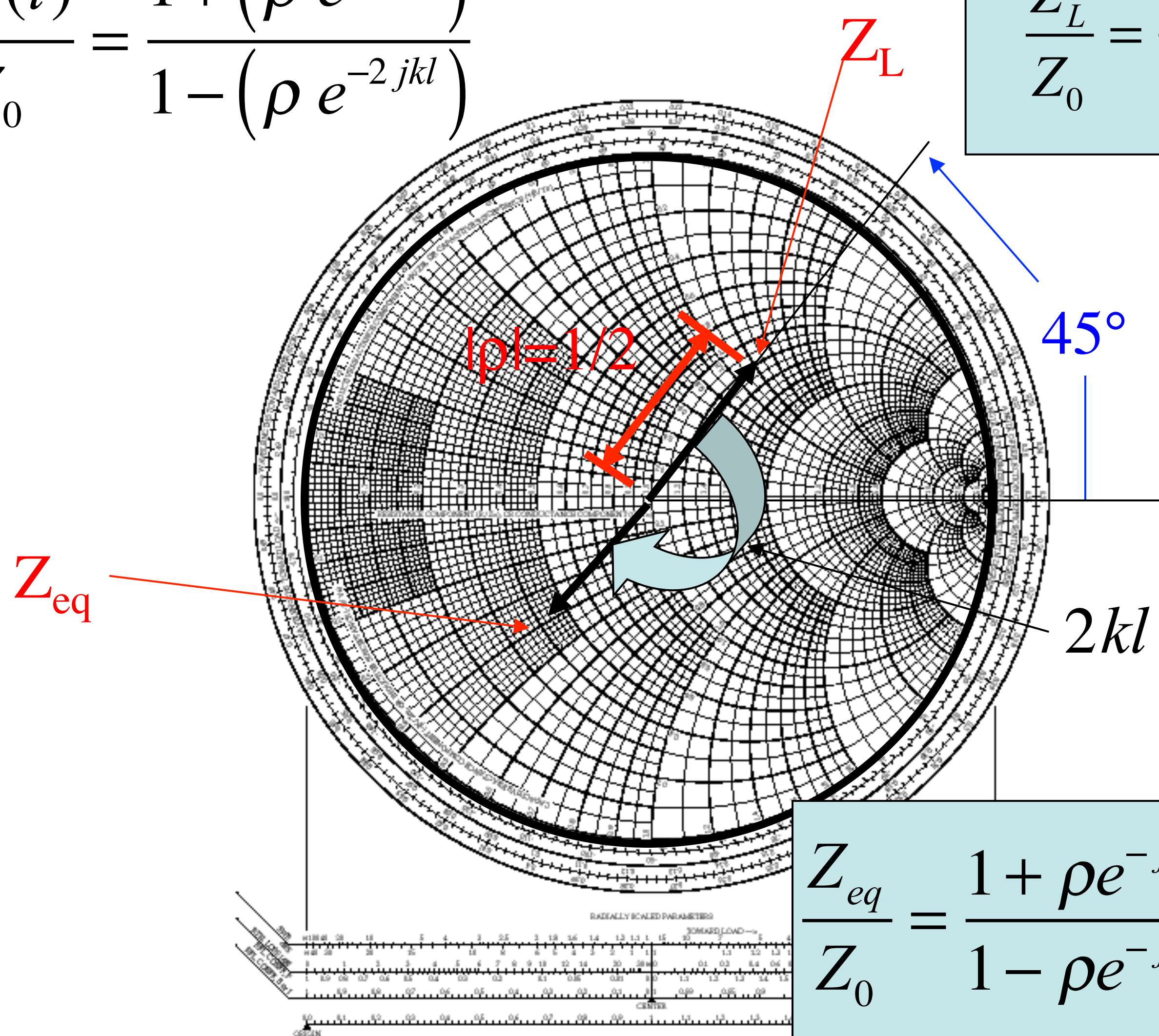


$$\frac{Z_L}{Z_0} = \frac{1+\rho}{1-\rho} = 1.4 + j1.3$$

What is Z_{eq} at $l = \lambda/4$ from the load?

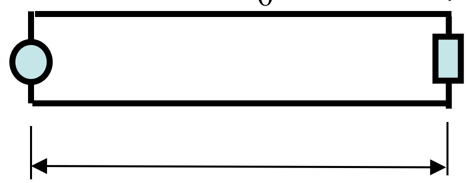
$$\frac{Z_{eq}(l)}{Z_0} = \frac{1 + \left(\rho\, e^{-2jkl}\right)}{1 - \left(\rho\, e^{-2jkl}\right)}$$

$$\frac{Z_L}{Z_0} = \frac{1+\rho}{1-\rho} = 1.4 + j1.3$$



$$\frac{Z_{eq}}{Z_0} = \frac{1+\rho e^{-j\pi}}{1-\rho e^{-j\pi}} = .38 - j.34$$

Sample Problem: find Z_{eq}



Method 1:

$$Z_{eq}(l) = Z_o \frac{Z_L \cos kl + jZ_0 \sin kl}{Z_0 \cos kl + jZ_L \sin kl}$$

$$k = 2\pi / \lambda$$

$$\lambda = v / f = 10^8 / 5 \times 10^6 = 20m$$

$$kl = 0.628$$

$$or$$

$$l / \lambda = .1$$

$$\cos kl = 0.81$$

$$\sin kl = 0.59$$

$$Z_{eq}(2) = Z_o \frac{56.63 + j99.5}{51.50 + j41.14}$$

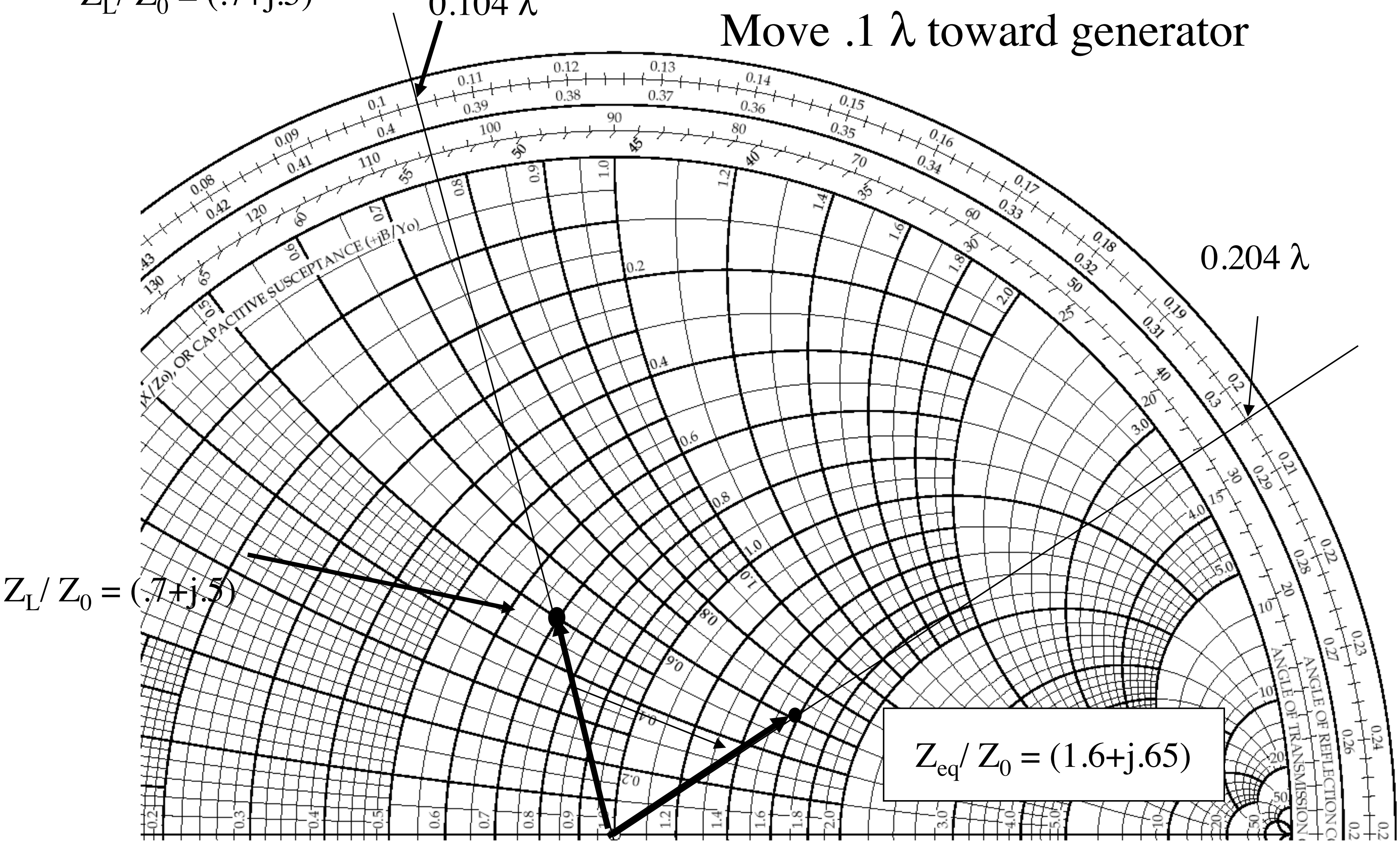
$$Z_{eq}(2) = (161 + j64)\Omega$$

Method 2: Smith Chart

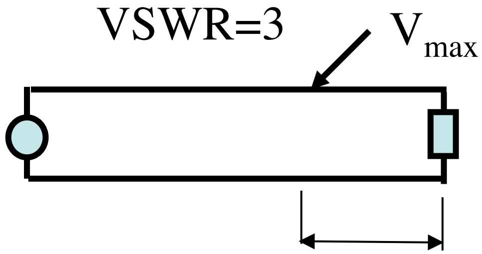
$Z_L / Z_0 = (70+j50)/100$

$Z_L / Z_0 = (.7+j.5)$

Move .1 λ toward generator



Standing Wave Problem

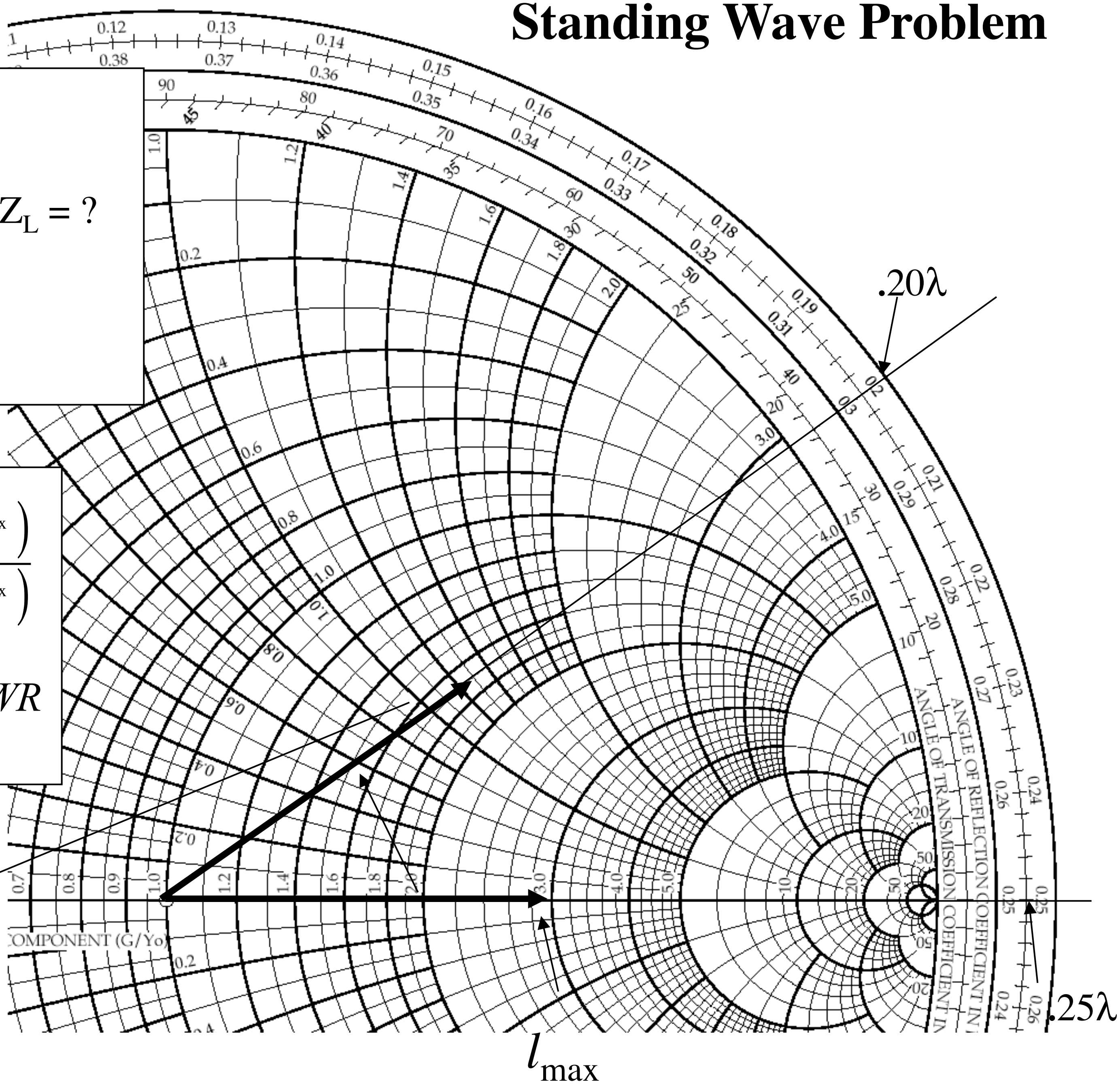


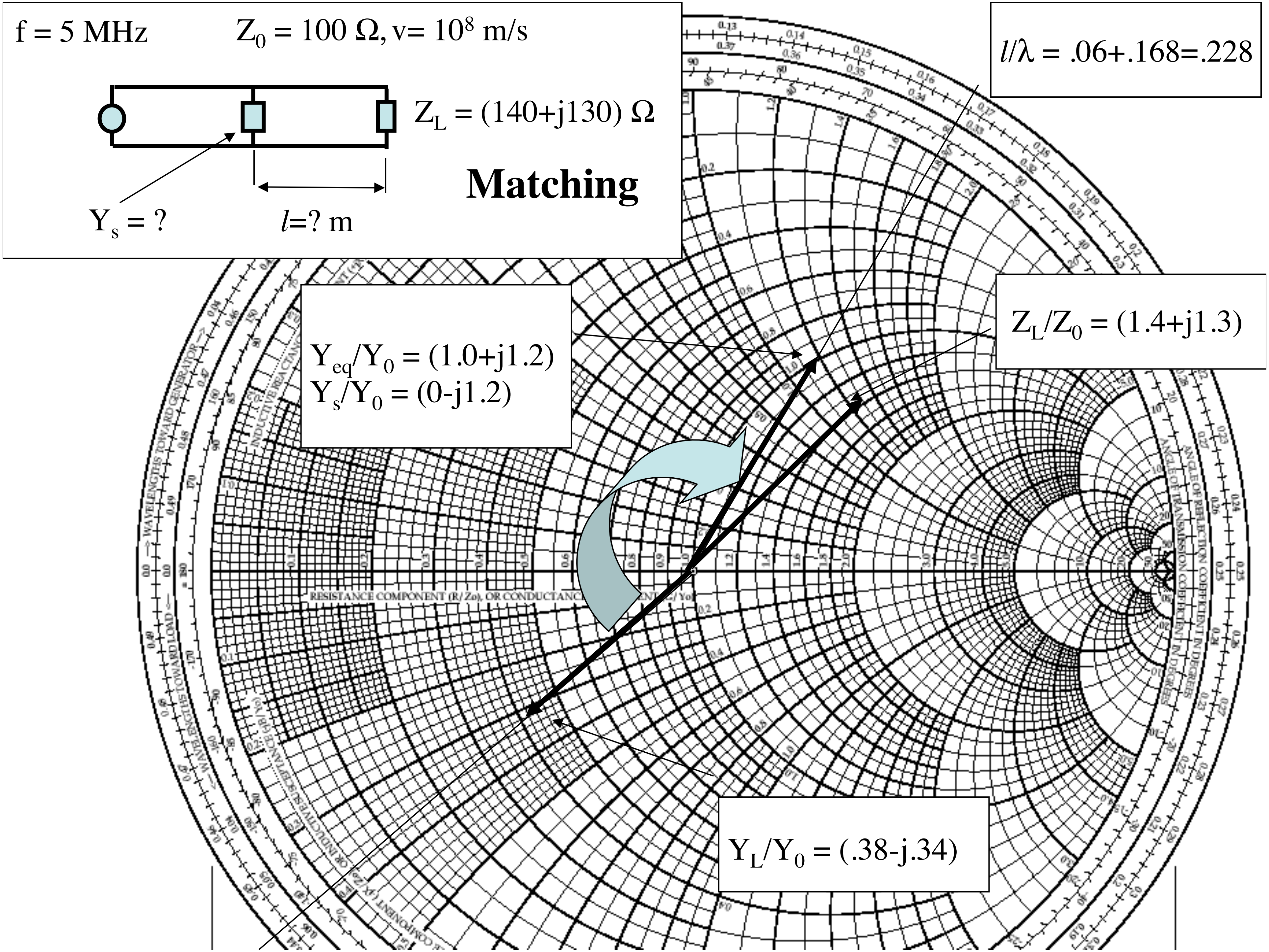
λ= 20m,
Z_0=100 Ω

$$\frac{Z_{eq}(l_{max})}{Z_0} = \frac{1+\left(\rho\, e^{-2jkl_{max}}\right)}{1-\left(\rho\, e^{-2jkl_{max}}\right)}$$

$$\frac{Z_{eq}(l_{max})}{Z_0} = \frac{1+|\rho|}{1-|\rho|} = VSWR$$

1.7+j1.3
Z_L=170+j130 Ω




f = 5 MHz
Z0 = 100 Ω, v= 10^8 m/s
ZL = (140+j130) Ω
Matching
Ys = ?
l=? m
l/λ = .06+.168=.228
ZL/Z0 = (1.4+j1.3)
Yeq/Y0 = (1.0+j1.2)
Ys/Y0 = (0-j1.2)
YL/Y0 = (.38-j.34)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES

Shunt admittance