General Comments

Tangential components of **E** are always equal.

Normal components of **B** are always equal.

Normal component of **E** discontinuous implies a **surface charge density.**

Normal components of **D** discontinuous implies a **free surface charge density**

Tangential components of **B** discontinuous implies a **surface current density.**

Tangential components of **H** discontinuous implies a **free surface current density.**

Boundary Condition Summary

Tangential Components

$$\mathbf{E}_{T2} = \mathbf{E}_{T1} \qquad \mathbf{H}_{T2} - \mathbf{H}_{T1} = \mathbf{K}_f \times \hat{\mathbf{n}} \qquad \mathbf{B}_{T2} - \mathbf{B}_{T1} = \mu_0 \mathbf{K}_t \times \hat{\mathbf{n}}$$

Normal Components

$$(\mathbf{D}_{N2} - \mathbf{D}_{N1}) \cdot \hat{\mathbf{n}} \Delta a = Q_f = \sigma_f \Delta a \quad \rightarrow \quad \boxed{\varepsilon_2 E_{N2} - \varepsilon_1 E_{N1} = \sigma_f}$$

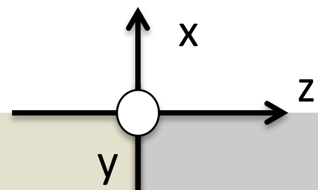
$$\varepsilon_0 (\mathbf{E}_{N2} - \mathbf{E}_{N1}) \cdot \hat{\mathbf{n}} \Delta a = Q_t = \sigma_t \Delta a \quad \rightarrow \quad \boxed{\varepsilon_0 E_{N2} - \varepsilon_0 E_{N1} = \sigma_t}$$

$$(\mathbf{B}_{N2} - \mathbf{B}_{N1}) \cdot \hat{\mathbf{n}} \Delta a = 0 \quad \rightarrow \quad \boxed{\mathbf{B}_{N2} = \mathbf{B}_{N1}}$$

$$\boxed{H_{N2} - H_{N1} = -(M_{N2} - M_{N1})}$$

Let's play the boundary condition game!

No free surface charge of current



Medium #1

$$\varepsilon_1 = 2\varepsilon_0, \mu_1 = \mu_0, \sigma_1 = 0$$

$$\mathbf{E}_1 = 2\hat{\mathbf{x}} + 4\hat{\mathbf{y}} + 6\hat{\mathbf{z}}$$

$$\mathbf{D}_1 / \varepsilon_0 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{B}_1 / \mu_0 = 6\hat{\mathbf{x}} + 7\hat{\mathbf{y}} + 8\hat{\mathbf{z}}$$

$$\mathbf{H}_1 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

Medium #2

$$\varepsilon_2 = 4\varepsilon_0, \mu_2 = 2\mu_0, \sigma_2 = 0$$

$$\mathbf{E}_2 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

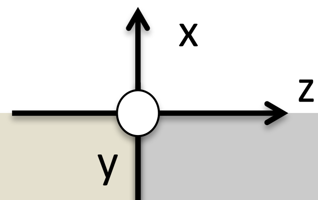
$$\mathbf{D}_2 / \varepsilon_0 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{B}_2 / \mu_0 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{H}_2 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

Let's play the boundary condition game!

No free surface charge of current



Medium #1

$$\varepsilon_1 = 2\varepsilon_0, \mu_1 = \mu_0, \sigma_1 = 0$$

$$\mathbf{E}_1 = 2\hat{\mathbf{x}} + 4\hat{\mathbf{y}} + 6\hat{\mathbf{z}}$$

$$\mathbf{D}_1 = \varepsilon_1 \mathbf{E}_1 = (2\varepsilon_0)\mathbf{E}_1$$

$$\mathbf{D}_1 / \varepsilon_0 = 2\mathbf{E}_1 = 4\hat{\mathbf{x}} + 8\hat{\mathbf{y}} + 12\hat{\mathbf{z}}$$

$$\mathbf{B}_1 / \mu_0 = 6\hat{\mathbf{x}} + 7\hat{\mathbf{y}} + 8\hat{\mathbf{z}}$$

$$\mathbf{H}_1 = \mathbf{B}_1 / \mu_1 = \mathbf{B}_1 / \mu_0$$

$$\mathbf{H}_1 = 6\hat{\mathbf{x}} + 7\hat{\mathbf{y}} + 8\hat{\mathbf{z}}$$

Medium #2

$$\varepsilon_2 = 4\varepsilon_0, \mu_2 = 2\mu_0, \sigma_2 = 0$$

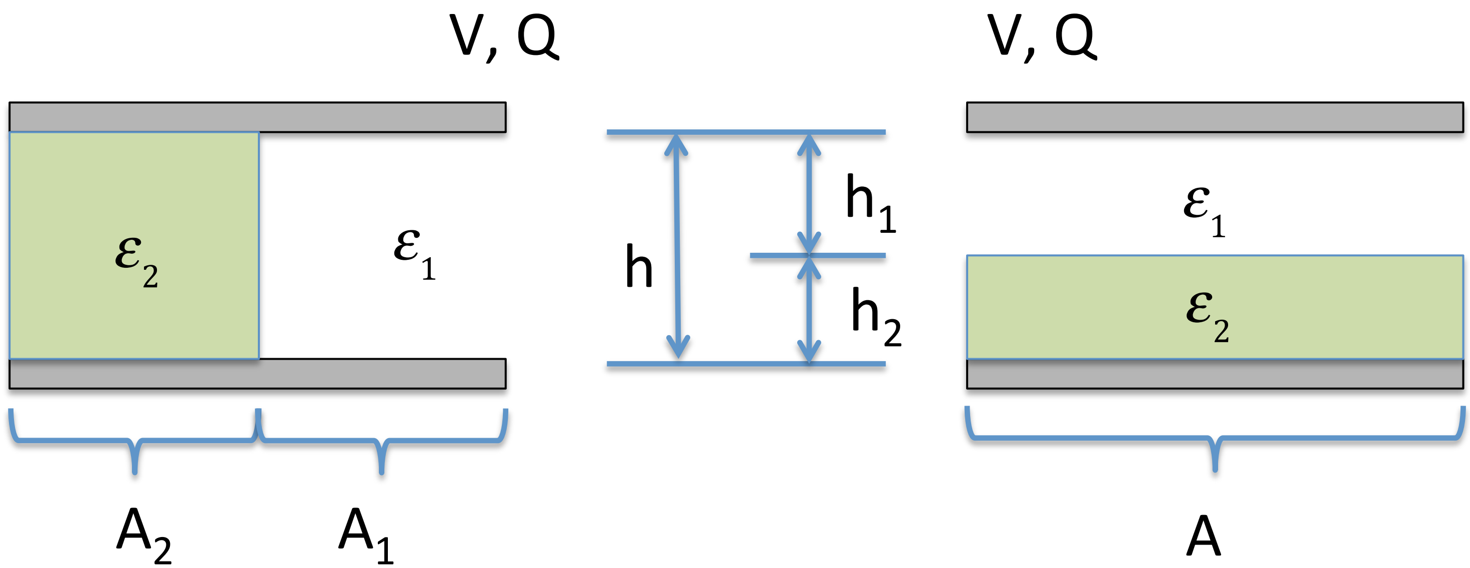
$$\mathbf{E}_2 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{D}_2 / \varepsilon_0 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{B}_2 / \mu_0 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

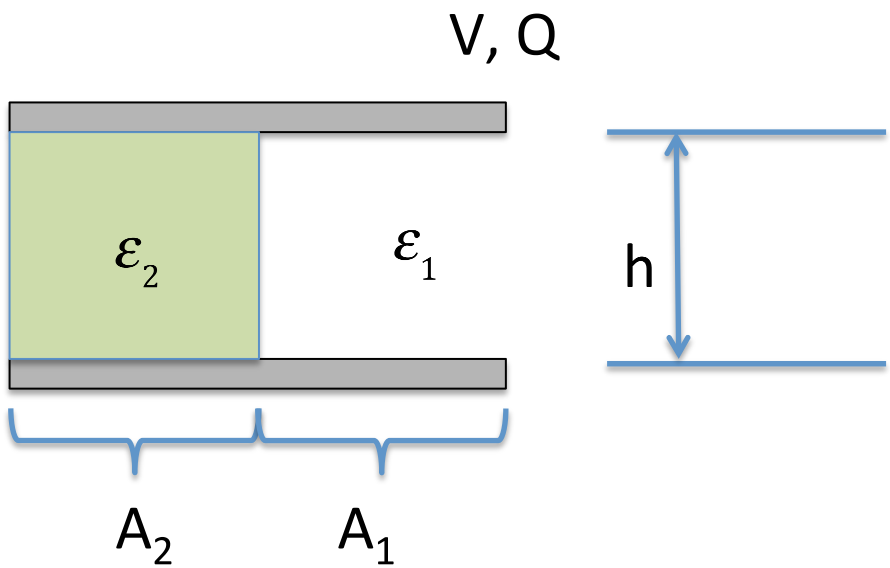
$$\mathbf{H}_2 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

Boundary Conditions in a Capacitor



Which boundary Conditions Apply?

Boundary Conditions in a Capacitor



Case #1

$E_1 = E_2 = V/h$ tangential E

$Q_1 = A_1 D_1 = A_1 \varepsilon_1 V/h$

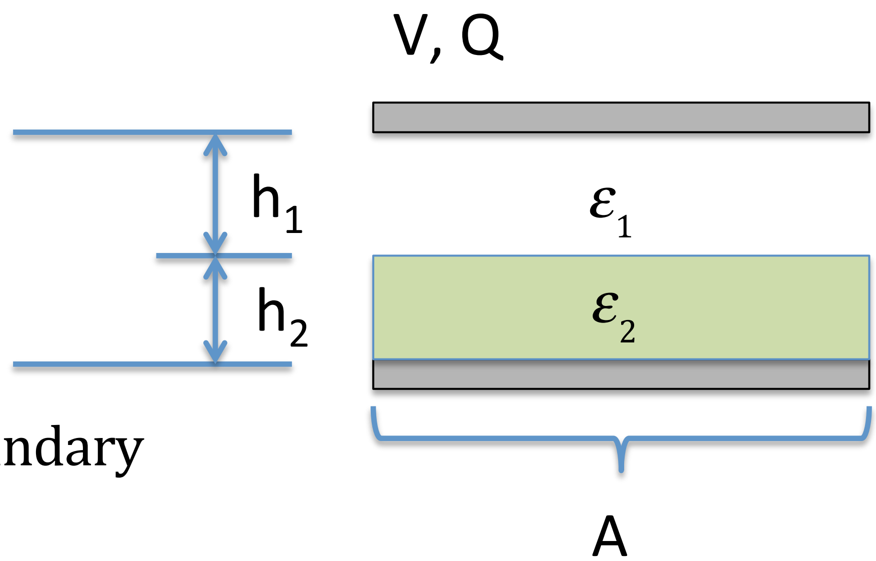
$Q_2 = A_2 D_2 = A_2 \varepsilon_2 V/h$

Case #1

$$Q = Q_1 + Q_2 = \left(\frac{A_1 \varepsilon_1 + A_2 \varepsilon_2}{h} \right) V$$

$$C = \left(\frac{A_1 \varepsilon_1 + A_2 \varepsilon_2}{h} \right)$$ Capacitors in parallel

Boundary Conditions in a Capacitor



Case #2

No free surface charge on boundary between ε_1 and ε_2.

$$D_1 = D_2 = Q/A$$

$$E_1 = D_1/\varepsilon_1 = Q/(\varepsilon_1 A)$$

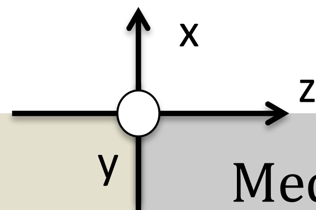
$$E_2 = D_2/\varepsilon_2 = Q/(\varepsilon_2 A)$$

Case #2

$$V = h_1 E_1 + h_2 E_2 = Q\left(\frac{h_1}{A\varepsilon_1} + \frac{h_2}{A\varepsilon_2}\right)$$

$$C^{-1} = \left(\frac{h_1}{A\varepsilon_1} + \frac{h_2}{A\varepsilon_2}\right)$$ Capacitors in series

Let's play the boundary condition game! With conductivity!!



Medium #1

$$\varepsilon_1 = 2\varepsilon_0, \mu_1 = \mu_0, \sigma_1 = \sigma$$

$$\mathbf{E}_1 = 2\hat{\mathbf{x}} + 4\hat{\mathbf{y}} + 6\hat{\mathbf{z}}$$

$$\mathbf{D}_1 / \varepsilon_0 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{B}_1 / \mu_0 = 6\hat{\mathbf{x}} + 7\hat{\mathbf{y}} + 8\hat{\mathbf{z}}$$

$$\mathbf{H}_1 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{J}_1 / \sigma = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

Medium #2

$$\varepsilon_2 = 4\varepsilon_0, \mu_2 = \mu_0, \sigma_2 = 2\sigma$$

$$\mathbf{E}_2 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{D}_2 / \varepsilon_0 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{B}_2 / \mu_0 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{H}_2 = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

$$\mathbf{J}_2 / \sigma = \quad \hat{\mathbf{x}} + \quad \hat{\mathbf{y}} + \quad \hat{\mathbf{z}}$$

ENEE381

Lecture 4

Conservation Laws

Charge, Energy, Momentum

Griffiths Chapter 8

Introduction

A consequence of the laws of Physics is that certain quantities are conserved once a closed system has been properly defined.

Some of these are:

Charge
Energy (and mass via $E=mc^2$)
Linear Momentum
Angular Momentum

Conservation Laws

Noether's Theorem

Conservation laws in physics are a direct consequence of **symmetries** in nature

Conservation of energy(mass) $\rightarrow$ time invariance

Conservation of linear momentum $\rightarrow$ translation invariance

Conservation of angular momentum $\rightarrow$ rotation invariance

Conservation of electric charge $\rightarrow$ gauge invariance (TBE)

Emmy Noether (Wikipedia)

Born	Amalie Emmy Noether 23 March 1882 Erlangen, Bavaria, German Empire
Died	14 April 1935 (aged 53) Bryn Mawr, Pennsylvania, United States
Nationality	German
Alma mater	University of Erlangen
Known for	Abstract algebra Theoretical physics Noether's theorem
Awards	Ackermann–Teubner Memorial Award (1932)
	Scientific career
Fields	Mathematics and physics
Institutions	University of Göttingen Bryn Mawr College
Thesis	*On Complete Systems of Invariants for Ternary Biquadratic Forms* (1907)

Example: conservation of kinetic + potential energy

$$\frac{d}{dt}m\mathbf{v} = q\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right]$$

Newton's law of motion (F=ma)

$$\text{Quasi}-\text{Static Fields:} \quad \mathbf{E} = -\nabla\Phi(\mathbf{x},t)$$

$$\mathbf{v}\cdot\frac{d}{dt}m\mathbf{v} = \frac{d}{dt}\frac{m\left|\mathbf{v}\right|^2}{2} = q\mathbf{v}\cdot\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right] = -q\mathbf{v}\cdot\nabla\Phi$$

Rate of change of potential following a trajectory

$$\frac{d}{dt}q\Phi(t,\mathbf{x}(t)) = \frac{\partial}{\partial t}q\Phi + q\mathbf{v}\cdot\nabla\Phi$$

$$\frac{d}{dt}\left(\frac{m\left|\mathbf{v}\right|^2}{2} + q\Phi\right) = \frac{\partial}{\partial t}q\Phi$$

Kinetic + Potential Energy is conserved only if potential is time independent

Conservation of Linear Momentum

$$\frac{d}{dt} m_i \mathbf{v}_i = q_i \mathbf{E}(\mathbf{x}_i, t) \qquad \mathbf{E}(\mathbf{x}_i, t) = \sum_{j \neq i} \frac{q_j \left(\mathbf{x}_i - \mathbf{x}_j\right)}{4\pi\varepsilon_0 \left|\mathbf{x}_i - \mathbf{x}_j\right|^3}$$

$$\frac{d}{dt} \sum_i m_i \mathbf{v}_i = \frac{d}{dt} \mathbf{P} = \sum_{i, j \neq i} \frac{q_i q_j \left(\mathbf{x}_i - \mathbf{x}_j\right)}{4\pi\varepsilon_0 \left|\mathbf{x}_i - \mathbf{x}_j\right|^3} = 0$$

Momentum **P** is constant,
velocity of center of mass is constant

$$\frac{d}{dt} \mathbf{X}_{cm} = \frac{d}{dt} \frac{\sum_i m_i \mathbf{x}_i}{\sum_i m_i} = \frac{\mathbf{P}}{M} = \text{constant}$$

If P =0, $\mathbf{X}_{cm}$ can not change

System is symmetric wrt translation in 3 directions. Three constants of motion: 3 components of **P**.

Conservation of Angular Momentum

$$\frac{d}{dt} m_i \mathbf{v}_i = q_i \mathbf{E}(\mathbf{x}_i, t) \qquad \mathbf{E}(\mathbf{x}_i, t) = \sum_{j \neq i} \frac{q_j \left(\mathbf{x}_i - \mathbf{x}_j\right)}{4\pi\varepsilon_0 \left|\mathbf{x}_i - \mathbf{x}_j\right|^3}$$

$$\frac{d}{dt}\mathbf{L} = \frac{d}{dt}\sum_i \mathbf{x}_i \times m_i \mathbf{v}_i = \sum_i \left(\frac{d\mathbf{x}_i}{dt} \times m_i \mathbf{v}_i + \mathbf{x}_i \times \frac{d}{dt} m_i \mathbf{v}_i \right)$$

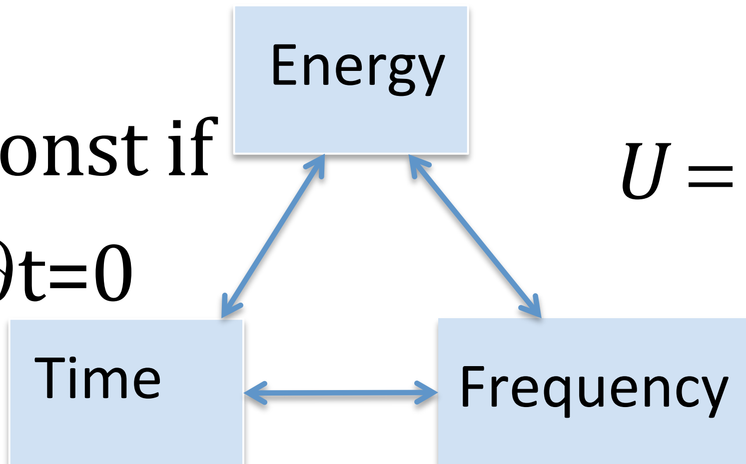
$$= \sum_{i, j \neq i} \mathbf{x}_i \times \frac{q_i q_j \left(\mathbf{x}_i - \mathbf{x}_j\right)}{4\pi\varepsilon_0 \left|\mathbf{x}_i - \mathbf{x}_j\right|^3} = 0$$

System is symmetric wrt rotation in 3 directions. Three constants of motion: 3 components of **L**.

Linked Quantities

U const if

$\partial/\partial t=0$

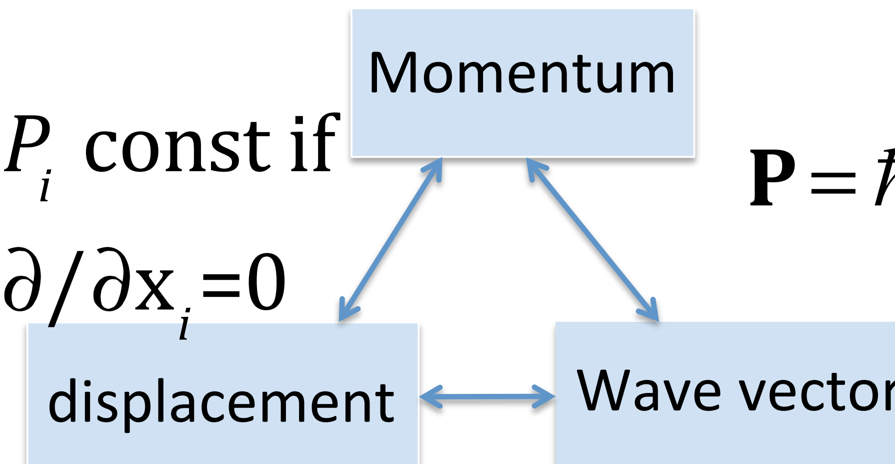


$$U=\hbar\omega$$

$$1=\Delta t\Delta\omega$$

P_i const if

$\partial/\partial x_i=0$



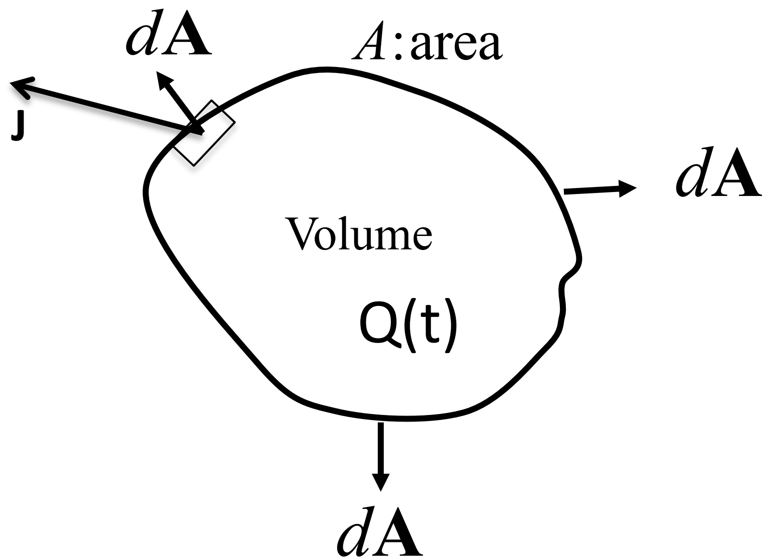
$$\mathbf{P}=\hbar\mathbf{k}$$

Sinusoidal waves

$$\exp\left(i\mathbf{k}\cdot\mathbf{x}-i\omega t\right)$$

$$1=\Delta x\Delta k$$

What does a conservation law for continuous systems look like?



$$\frac{dQ}{dt} + \int_S d\vec{\mathbf{A}} \cdot \vec{\mathbf{J}} = 0$$

$$Q = \int_V d^3 r \rho(\mathbf{r}, t)$$

$$\int_S d\vec{\mathbf{A}} \cdot \vec{\mathbf{J}} = \int_V d^3 r \, \nabla \cdot \vec{\mathbf{J}}$$

Conservation of charge

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{\mathbf{J}} = 0$$

Conservation of Energy

$$\frac{\partial}{\partial t}\left[u_E + u_M\right] + \nabla \cdot \vec{\mathbf{S}} = -\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}$$

Rate at which energy is transferred to current J

$$u_E + u_M = \frac{\varepsilon_0}{2}\mathbf{E} \cdot \mathbf{E} + \frac{1}{2\mu_0}\mathbf{B} \cdot \mathbf{B}$$

Energy density in fields

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} : \text{ Poynting vector}$$

Flow of local energy density

Conservation of energy

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \nabla \times \vec{\mathbf{B}} = \mu_0 \left[\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right]$$

$$\frac{\vec{\mathbf{B}}}{\mu_0} \cdot \nabla \times \mathbf{E} = -\frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \mathbf{E} \cdot \nabla \times \frac{\vec{\mathbf{B}}}{\mu_0} = \left[\vec{\mathbf{E}} \cdot \vec{\mathbf{J}} \cdot + \varepsilon_0 \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial t} \right]$$

$$\varepsilon_0 \vec{\mathbf{E}} \cdot \frac{\partial \vec{\mathbf{E}}}{\partial t} + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t} + \frac{\vec{\mathbf{B}}}{\mu_0} \cdot \nabla \times \mathbf{E} - \mathbf{E} \cdot \nabla \times \frac{\vec{\mathbf{B}}}{\mu_0} = -\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}$$

$$\frac{\partial}{\partial t} \left(\frac{\varepsilon_0 \left| \vec{\mathbf{E}} \right|^2}{2} + \frac{\left| \vec{\mathbf{B}} \right|^2}{2\mu_0} \right) + \nabla \cdot \left(\mathbf{E} \times \frac{\vec{\mathbf{B}}}{\mu_0} \right) = -\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}$$

Poynting's Theorem

$$\frac{\partial}{\partial t}\left(\frac{\varepsilon_0|\mathbf{E}|^2}{2}+\frac{\mu_0|\mathbf{H}|^2}{2}\right)+\nabla\cdot(\mathbf{E}\times\mathbf{H})=-\mathbf{E}\cdot\mathbf{J}$$

Energy density

$$\left(\frac{\varepsilon_0|\mathbf{E}|^2}{2}+\frac{\mu_0|\mathbf{H}|^2}{2}\right)$$

Power Flux

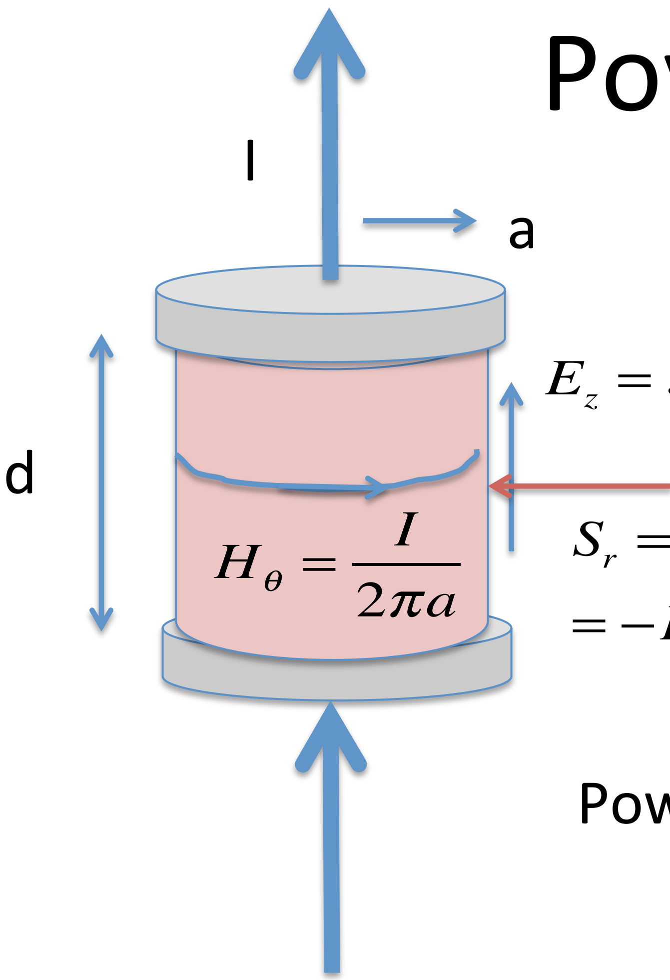
$$\mathbf{S}=(\mathbf{E}\times\mathbf{H})$$

Rate of work done by E on J

$$\mathbf{E}\cdot\mathbf{J}$$

Units: Joules/m^3 Watts/m^2 Watts/m^3

Poynting Example



$$E_z = J_z / \sigma = I / (\pi a^2 \sigma)$$

$$\vec{\mathbf{S}} = \left(\vec{\mathbf{E}} \times \vec{\mathbf{H}}\right)$$

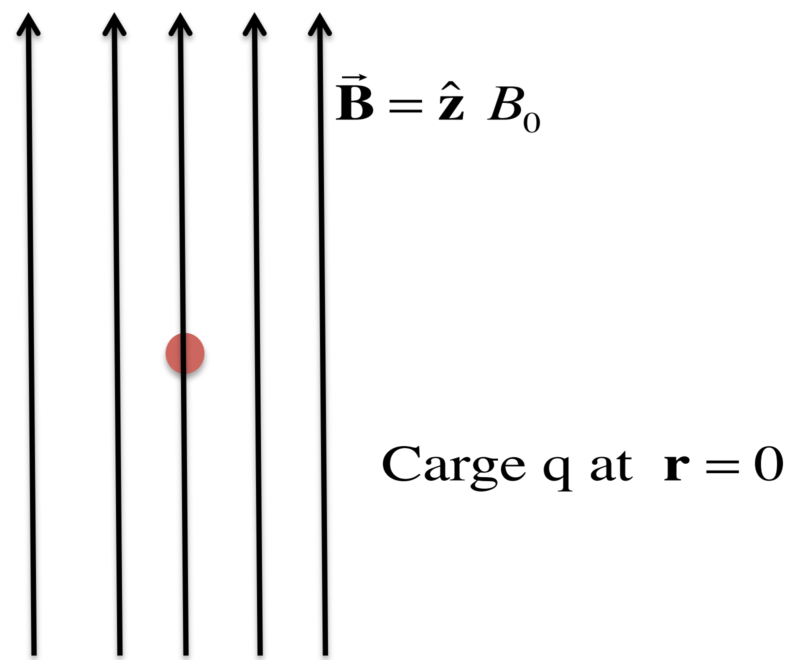
$$S_r = -E_z H_\theta$$
$$= -I^2 / (2\pi^2 a^3 \sigma)$$

Area of side

Power in: $P = 2\pi a d |S_r| =$
$$= I^2 / (\pi\ a^2 \sigma) = R I^2$$

Resistance

Only divergence of Poynting flux matters



Find S:
What direction?
What does it mean?

Poynting's theorem addresses EM energy, what about mechanical energy?

$$\frac{\partial}{\partial t}\left(\frac{\varepsilon_0|\mathbf{E}|^2}{2}+\frac{\mu_0|\mathbf{H}|^2}{2}\right)+\nabla\cdot(\mathbf{E}\times\mathbf{H})=-\mathbf{E}\cdot\mathbf{J}$$

Rate of work done by E on J

Newton's Law ma=F

$$m\frac{d}{dt}\mathbf{v}_i=q[\mathbf{E}+\mathbf{v}_i\times\mathbf{B}]$$

$$m\sum_i \mathbf{v}_i\cdot\frac{d}{dt}\mathbf{v}_i=\sum_i \mathbf{v}_i\cdot q[\mathbf{E}+\mathbf{v}_i\times\mathbf{B}]=\sum_i \mathbf{v}_i\cdot q\mathbf{E}=\int_V d^3r\,\mathbf{v}_i\cdot q\mathbf{E}$$

$$\sum_i \frac{d}{dt}\frac{m|\mathbf{v}_i|^2}{2}=\int_V d^3r\,\mathbf{v}_i\cdot q\mathbf{E}=\int_V d^3r\,\mathbf{J}\cdot\mathbf{E}$$

Combining EM and Mechanical Energy

$$\frac{d}{dt}\left\{\int_V d^3r\left(\frac{\varepsilon_0|\mathbf{E}|^2}{2}+\frac{\mu_0|\mathbf{H}|^2}{2}\right)+\sum_i \frac{m|\mathbf{v}_i|^2}{2}\right\}+\int_S d\mathbf{A}\cdot(\mathbf{E}\times\mathbf{H})=0$$

EM + Mechanical Energy EM power flow

Conservation of EM Momentum

The total EM force on charges in a volume can be written as

$$\frac{d\mathbf{P}_{mech}}{dt} = \sum_i q(\mathbf{E}(\mathbf{x}_i) + \mathbf{v}_i \times \mathbf{B}(\mathbf{x}_i)) = \int_V (\rho\mathbf{E} + \mathbf{J}\times\mathbf{B})d^3r$$

After some Math $\quad \frac{d\mathbf{P}_{mech}}{dt} + \frac{d\mathbf{P}_{EM}}{dt} = \oint_A \overline{\overline{\mathbf{T}}} \cdot \hat{\mathbf{n}}da$

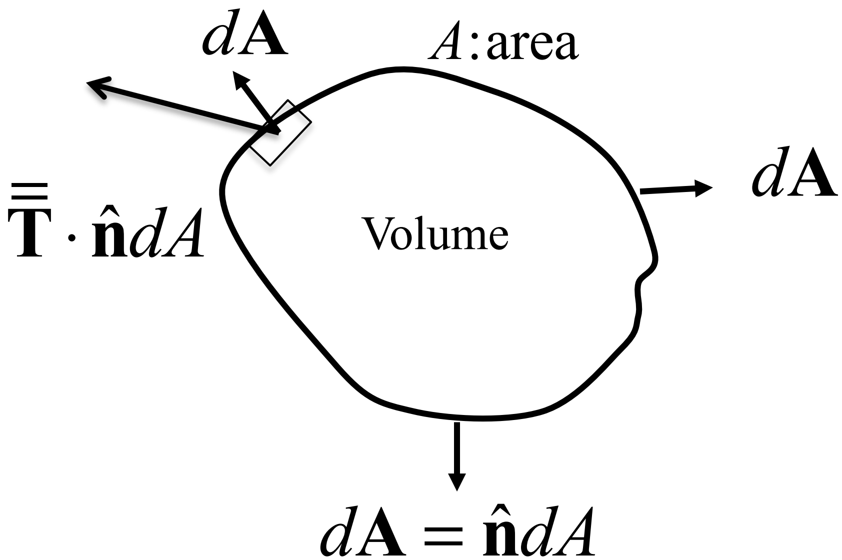
Total EM linear momentum: $\mathbf{P}_{EM} = \varepsilon_0\mu_0 \int_V \mathbf{E}\times\mathbf{H}\, d^3r$

EM linear momentum density: $\varepsilon_0\mu_0\mathbf{E}\times\mathbf{H} = \mathbf{S}/c^2$

Poynting vector: $\mathbf{S} = \mathbf{E}\times\mathbf{H}, \quad \mu_0\varepsilon_0 = 1/c^2$

Maxwell Stress Tensor: $\overline{\overline{\mathbf{T}}} = \varepsilon_0\mathbf{E}\mathbf{E} + \frac{1}{\mu_0}\mathbf{B}\mathbf{B} - \frac{1}{2}(\varepsilon_0\mathbf{E}\cdot\mathbf{E} + \frac{1}{\mu_0}\mathbf{B}\cdot\mathbf{B})\overline{\overline{\mathbf{I}}}$

Force on what's inside



$$\frac{d\mathbf{P}_{mech}}{dt} + \frac{d\mathbf{P}_{EM}}{dt} = \mathbf{F}$$

$$\mathbf{F} = \oint_A \bar{\bar{\mathbf{T}}} \cdot \hat{\mathbf{n}} dA$$

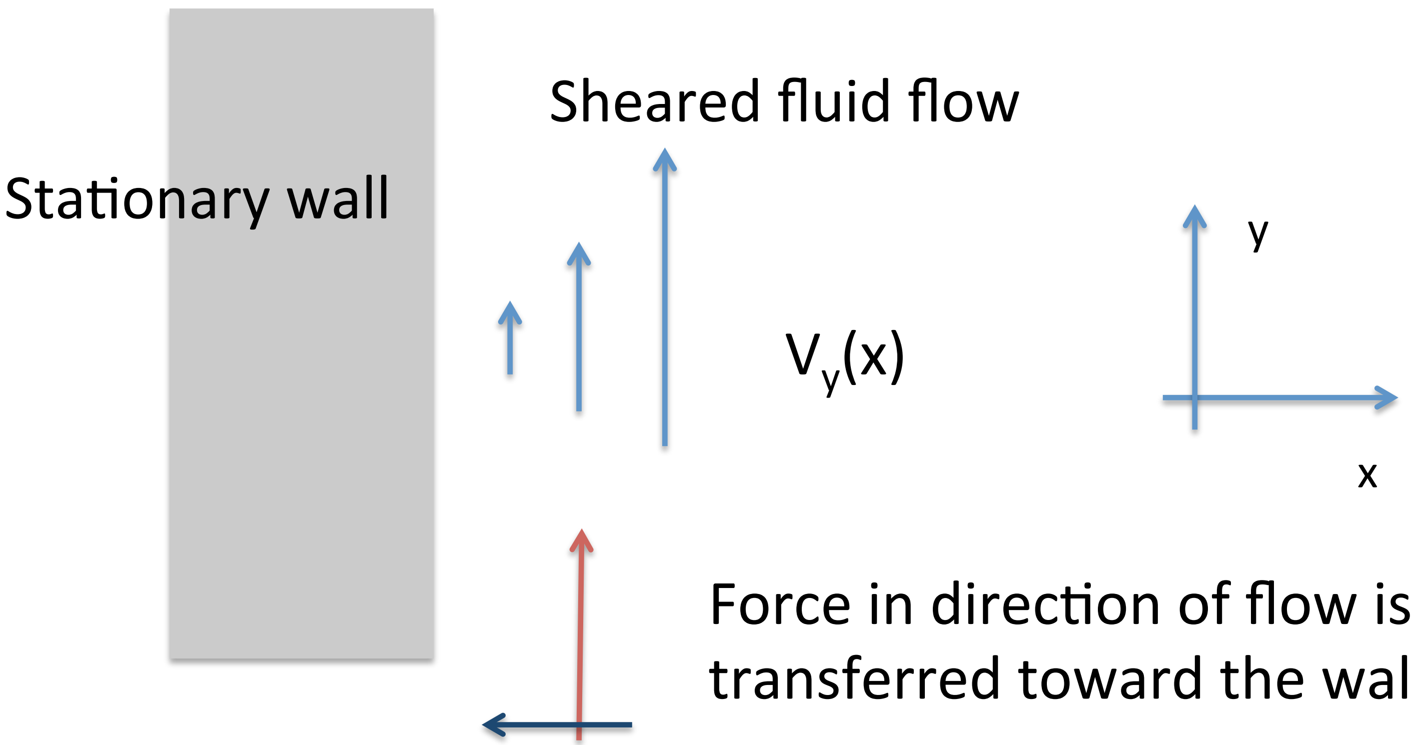
Analogy: pressure

$$\bar{\bar{\mathbf{T}}} = -p \bar{\bar{\mathbf{I}}}$$

$$\bar{\bar{\mathbf{T}}} \cdot \hat{\mathbf{n}} = -p \hat{\mathbf{n}}$$

$$\text{Maxwell Stress Tensor: } \bar{\bar{\mathbf{T}}} = \varepsilon_0 \mathbf{E}\mathbf{E} + \frac{1}{\mu_0} \mathbf{B}\mathbf{B} - \frac{1}{2}\left(\varepsilon_0 \mathbf{E} \cdot \mathbf{E} + \frac{1}{\mu_0} \mathbf{B} \cdot \mathbf{B}\right) \bar{\bar{\mathbf{I}}}$$

Viscous Fluid Stress



Force in direction of flow is transferred toward the wall

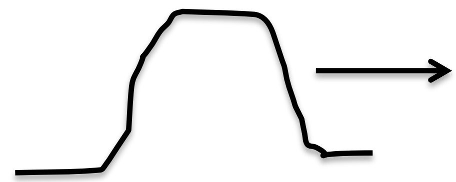
$$T_{xy} \propto \nu \frac{\partial v_y(x)}{\partial x}$$

Energy and Momentum of Light

$$\left(\frac{\varepsilon_0 |\mathbf{E}|^2}{2} + \frac{\mu_0 |\mathbf{H}|^2}{2} \right) \qquad \vec{\mathbf{S}} = \left(\vec{\mathbf{E}} \times \vec{\mathbf{H}} \right)$$

Energy density   Power Flux

Units: Joules/m^3 Watts/m^2



Pulse
Moves at c

Power Flux = c Energy Density

Pulse also contains momentum

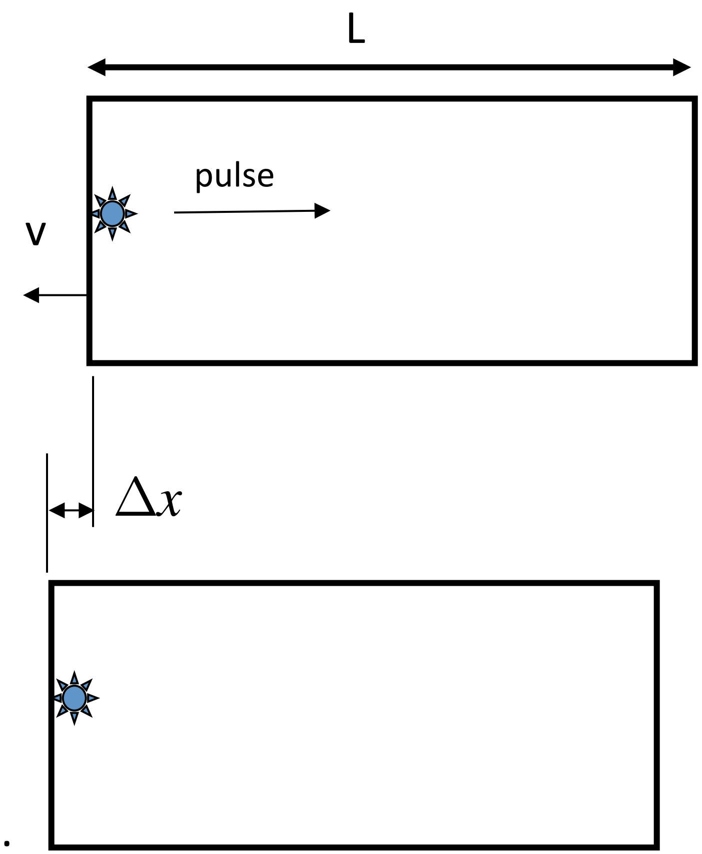
$$\text{EM linear momentum density:} \quad \varepsilon_0 \mu_0 \mathbf{E} \times \mathbf{H} = \mathbf{S} / c^2$$

$$\frac{\text{Energy Density}}{\text{Momentum Density}} = \frac{S / c}{S / c^2} = c$$

A pulse of light carries energy and momentum: ratio = c

Mass Energy Equivalence $E = mc^2$

Isolated box of mass M and length L in space.
A light on the wall on one side sends out a pulse of energy E toward the right.
The pulse has momentum p=E/c.
The box recoils with velocity v=p/M to the left.
The pulse is absorbed on the other side after a time T=L/c.
The box absorbs the momentum and stops moving.



Displacement of the box $\Delta x = vT = \frac{EL}{Mc^2}$

Has the center of mass moved?
We would like to say no.
The box should not be able to move its center of mass.

$$\Delta x M = L\left(E / c^2\right) = Lm$$

We can say that the CM has not moved if the pulse reduced the mass of the left side by $m=E/c^2$ and increased the right side by the same amount.

$$E = mc^2$$

Stress Tensor

$$\overline{\overline{\mathbf{T}}} = \varepsilon_0 \mathbf{E}\mathbf{E} + \frac{1}{\mu_0}\mathbf{B}\mathbf{B} - \frac{1}{2}(\varepsilon_0 \mathbf{E}\cdot\mathbf{E} + \frac{1}{\mu_0}\mathbf{B}\cdot\mathbf{B})\overline{\overline{\mathbf{I}}}$$

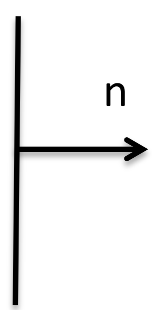
Force transmitted through surface $\mathbf{F} = \oint_A \overline{\overline{\mathbf{T}}}\cdot\hat{\mathbf{n}}\,da$

The component normal to the surface is like a pressure force $\mathbf{n}\cdot\overline{\overline{\mathbf{T}}}\cdot\mathbf{n} = -p$

$$\mathbf{n}\cdot\overline{\overline{\mathbf{T}}}\cdot\mathbf{n} = \varepsilon_0\left[\frac{1}{2}\left(\mathbf{n}\cdot\mathbf{E}\right)^2 - \frac{1}{2}\left|\mathbf{E}_t\right|^2\right] + \frac{1}{\mu_0}\left[\frac{1}{2}\left(\mathbf{n}\cdot\mathbf{B}\right)^2 - \frac{1}{2}\left|\mathbf{B}_t\right|^2\right]$$

Remember BC's
E_t and B_n are continuous

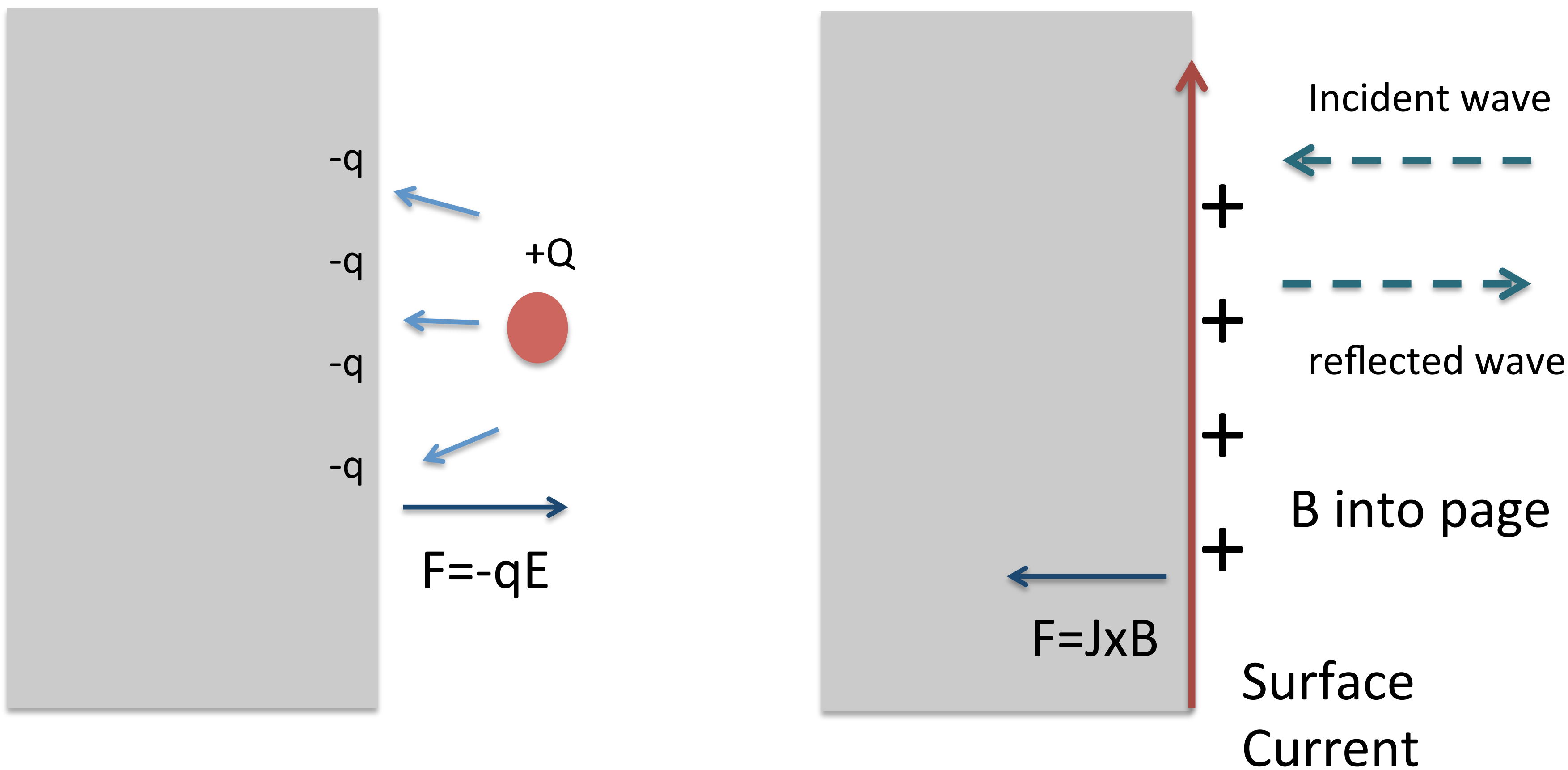
Normal E pulls on surface
Tangential B pushes



Surface of conductor

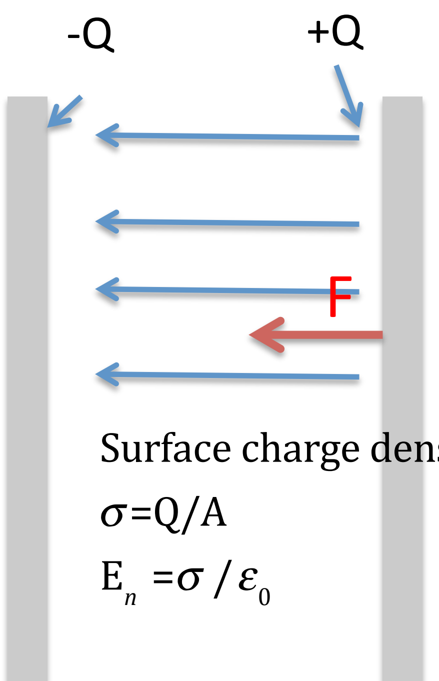
$$\mathbf{n}\cdot\overline{\overline{\mathbf{T}}}\cdot\mathbf{n} = \varepsilon_0\left[\frac{1}{2}\left(\mathbf{n}\cdot\mathbf{E}\right)^2\right] + \frac{1}{\mu_0}\left[-\frac{1}{2}\left|\mathbf{B}_t\right|^2\right]$$

Forces on Conductor



Electric field force on surface charge pulls

Force of attraction between capacitor plates



Area = A

$$\mathbf{F} = \oint_A \overline{\overline{\mathbf{T}}} \cdot \hat{\mathbf{n}}\, da$$

$$\mathbf{n} \cdot \overline{\overline{\mathbf{T}}} \cdot \mathbf{n} = \varepsilon_0 \left[\frac{1}{2} \left(\mathbf{n} \cdot \mathbf{E} \right)^2 \right] = \frac{1}{2} \frac{Q^2}{A^2 \varepsilon_0}$$

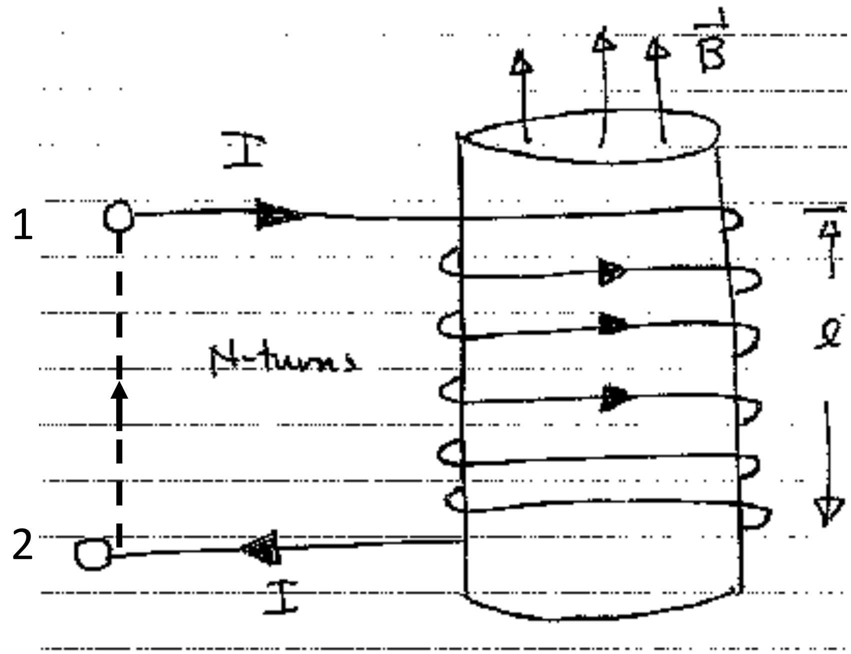
$$F = \frac{1}{2} \frac{Q^2}{A\ \varepsilon_0}$$

How much work must be done to separate plates a distance h?

$$\text{Work} = hF = \frac{h}{2} \frac{Q^2}{A\ \varepsilon_0} = \frac{1}{2} \frac{Q^2}{C}$$

capacitance

What is the force on the windings of a coil?



$$\mathbf{n}\cdot\overline{\overline{\mathbf{T}}}\cdot\mathbf{n}=\varepsilon_0\left[\frac{1}{2}(\mathbf{n}\cdot\mathbf{E})^2-\frac{1}{2}|\mathbf{E}_t|^2\right]+\frac{1}{\mu_0}\left[\frac{1}{2}(\mathbf{n}\cdot\mathbf{B})^2-\frac{1}{2}|\mathbf{B}_t|^2\right]$$

Energy Density in a Linear Medium

Field Energy

Energy density

$$\left(\frac{\varepsilon_0\left|\vec{\mathbf{E}}\right|^2}{2}+\frac{\left|\vec{\mathbf{B}}\right|^2}{2\mu_0}\right)$$

Power Flux

$$\vec{\mathbf{S}}=\left(\vec{\mathbf{E}}\times\vec{\mathbf{H}}\right)$$

Rate of work done by E on J

$$\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}$$

$$\vec{\mathbf{B}}=\mu\vec{\mathbf{H}} \qquad \vec{\mathbf{D}}=\varepsilon\vec{\mathbf{E}}$$

Energy density

$$\left(\frac{\varepsilon\left|\vec{\mathbf{E}}\right|^2}{2}+\frac{\mu\left|\vec{\mathbf{H}}\right|^2}{2}\right)$$

Power Flux

$$\vec{\mathbf{S}}=\left(\vec{\mathbf{E}}\times\vec{\mathbf{H}}\right)$$

Rate of work done by E on J

$$\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}$$

Almost always wrong

$$\frac{\partial}{\partial\omega}\left(\frac{\omega\varepsilon\left|\vec{\mathbf{E}}\right|^2}{2}+\frac{\omega\mu\left|\vec{\mathbf{H}}\right|^2}{2}\right)$$