ENEE381

Lecture 3
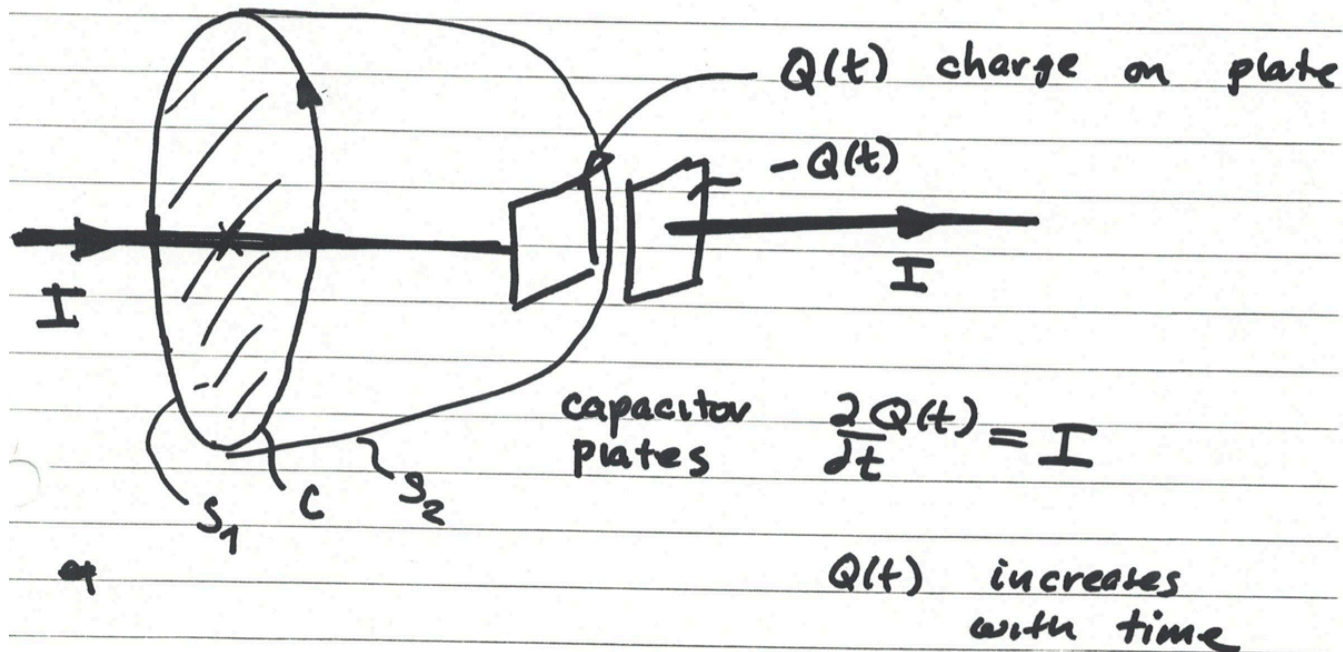Displacement Current
Fields in Matter
Boundary Conditions

Maxwell's Displacement Current

It is not really a current. It just acts like one.

Maxwell determined the static Ampere's Law could not be correct.
Inconsistent with charge conservation

$$\oint_{Loop} \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{l}} = \mu_0 \int_S d\vec{\mathbf{A}} \cdot \vec{\mathbf{J}} = \mu_0 I$$

CONSIDER THE FOLLOWING EXAMPLE

Q(t) charge on plate

-Q(t)

I

$$\int_{S_1} d\vec{\mathbf{A}} \cdot \vec{\mathbf{J}} = I$$

$$\int_{S_2} d\vec{\mathbf{A}} \cdot \vec{\mathbf{J}} = 0$$

capacitor
Plates

$$\frac{\partial Q(t)}{\partial t} = I$$

I

S_1 C S_2

Q(t) increases
with time

Remember for Faraday's Law
Any surface with the same
perimeter gave the correct
answer.

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\int_S d\vec{\mathbf{A}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

From Gauss' Law

$$\int_{S_1+S_2} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$d\vec{\mathbf{S}}_1 = d\vec{\mathbf{S}}$$

$$d\vec{\mathbf{S}}_2 = -d\vec{\mathbf{S}}$$

$$\int_{S_1+S_2} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = 0 \Rightarrow \int_{S_1} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}}_1 = \int_{S_2} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}}_2$$
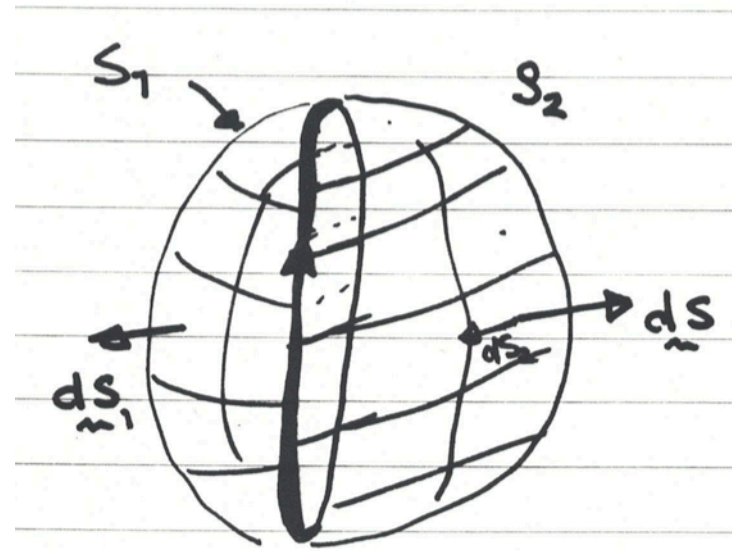
Conservation of Charge
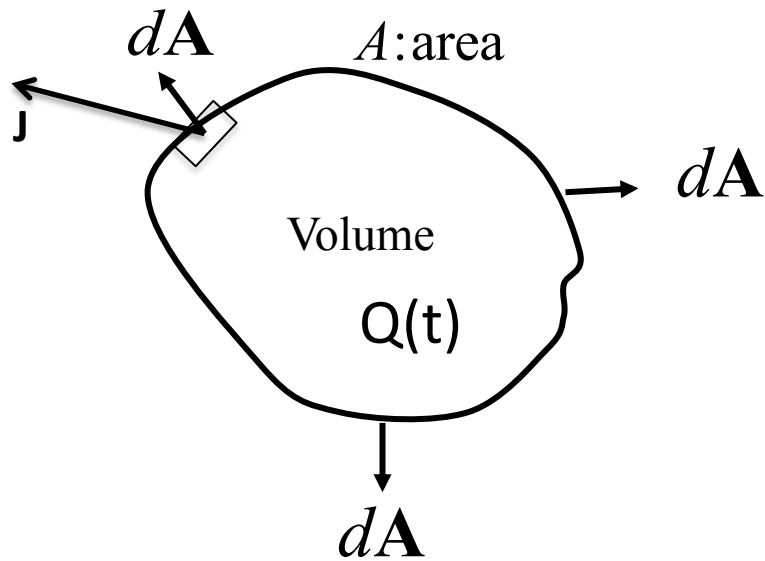
$$\int_S d\vec{\mathbf{A}} \cdot \vec{\mathbf{J}} + \frac{dQ}{dt} = 0$$
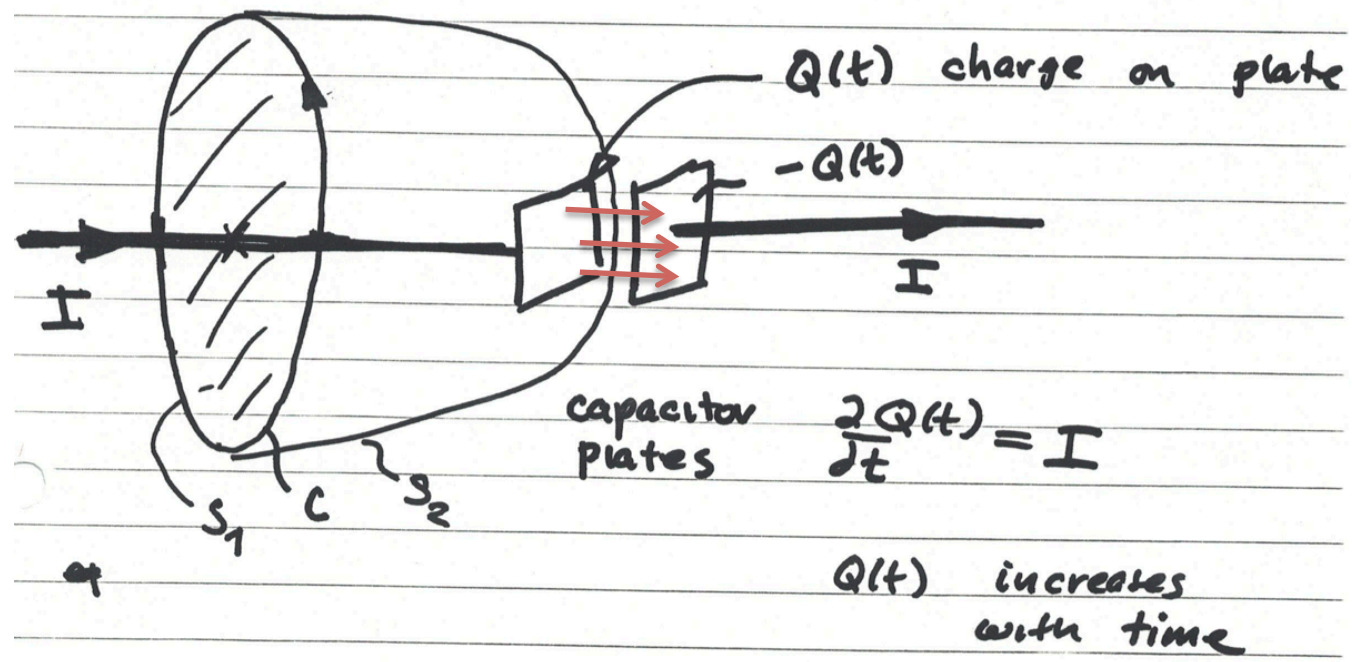
But

$$\oint_S \varepsilon_0 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = Q$$

So

$$\int_S d\vec{\mathbf{A}} \cdot \left(\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial}{\partial t} \vec{\mathbf{E}} \right) = 0$$

CONSIDER THE FOLLOWING EXAMPLE

$Q(t)$ charge on plate

$-Q(t)$

I

I

capacitor plates $\quad \frac{\partial}{\partial t} Q(t) = I$

$Q(t)$ increases with time

$S_1 \quad C \quad S_2$

$$\int\limits_{S} d\vec{\mathbf{A}} \cdot \left(\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial}{\partial t} \vec{\mathbf{E}} \right) = 0$$

Faraday: time varying B makes an E

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} == -\frac{d}{dt} \int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

example

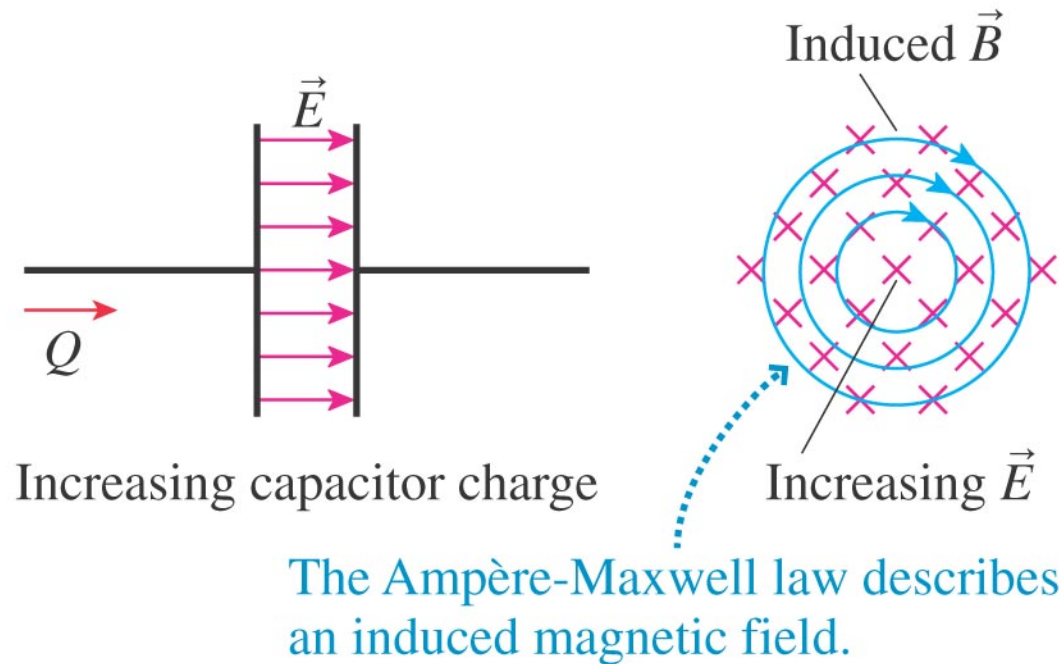$$E_\theta(r) = -\frac{r}{2} \frac{\partial B_z}{\partial t}$$
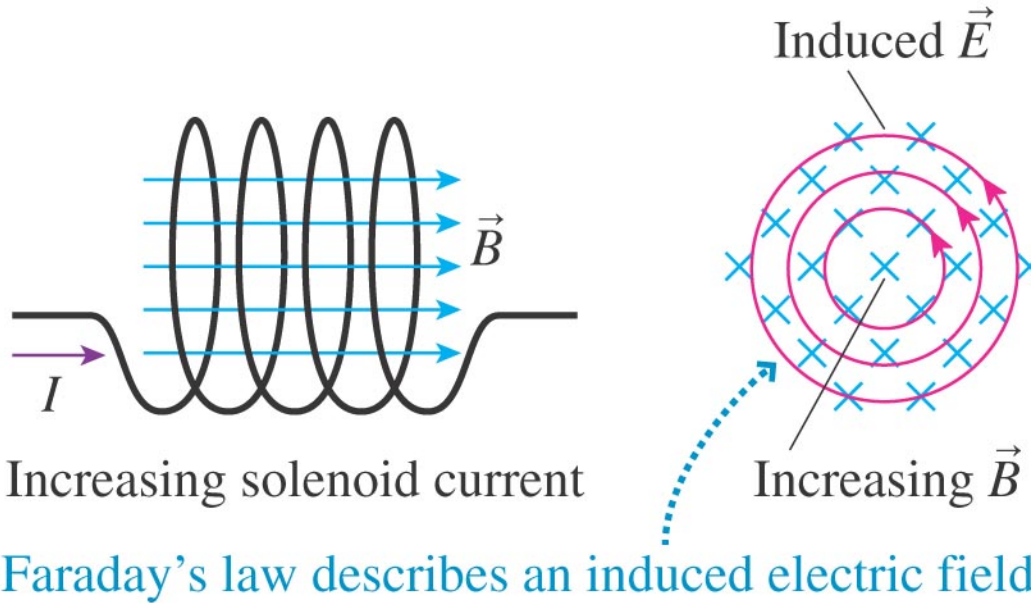
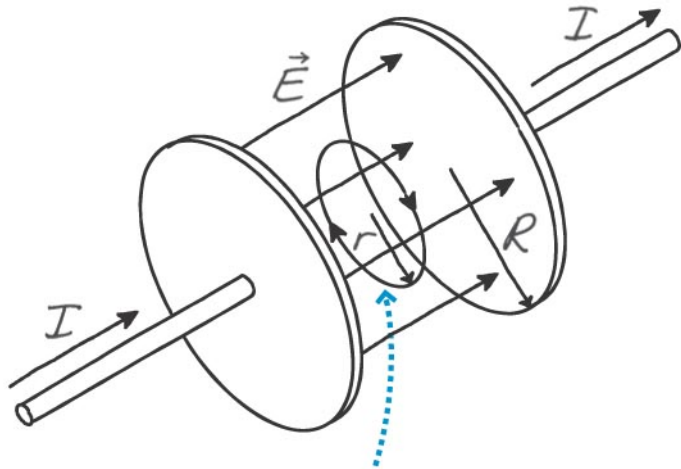Ampere-Maxwell: time varying E makes a B

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

example

$$B_\theta(r) = \frac{\mu_0 \varepsilon_0 r}{2} \frac{\partial E_z}{\partial t}$$

Put together, fields can sustain themselves - Electromagnetic Waves

Induced $\vec{E}$

$\vec{B}$

I

Increasing solenoid current

Increasing $\vec{B}$

Faraday's law describes an induced electric field.

Induced $\vec{B}$

$\vec{E}$

Q

Increasing capacitor charge

Increasing $\vec{E}$

The Ampère-Maxwell law describes
an induced magnetic field.

The magnetic field line is a circle concentric with the capacitor. The electric flux through this circle is $\pi r^2 E$.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = B_\theta(r) 2\pi r$$

$$\Phi_e = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \pi r^2 E_z$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 \left(I + \varepsilon_0 \frac{d\Phi_e}{dt} \right)$$

$$B_\theta(r) = \frac{\mu_0 \varepsilon_0 r}{2} \frac{\partial E_z}{\partial t}$$

Recall from Faraday:

$$E_\theta(r) = -\frac{r}{2} \frac{\partial B_z}{\partial t}$$

Conduction current, Displacement current, Polarization current

In vacuum $\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \right)$

Flow of charge "real" current

Displacement "current"

$$\mathbf{J} = \mathbf{J}_f + \mathbf{J}_p$$

"Free" current

polarization current $\qquad \mathbf{J}_p = \varepsilon_0 \chi_E \dfrac{\partial \mathbf{E}}{\partial t}$

$$\mathbf{J}_p = \dfrac{\partial}{\partial t} \left[\varepsilon_0 \chi_E \mathbf{E} \right]$$

Electric Susceptibility χ_E

$E = 0$

neutral molecular

$E \neq 0$

molecules become "polarized"
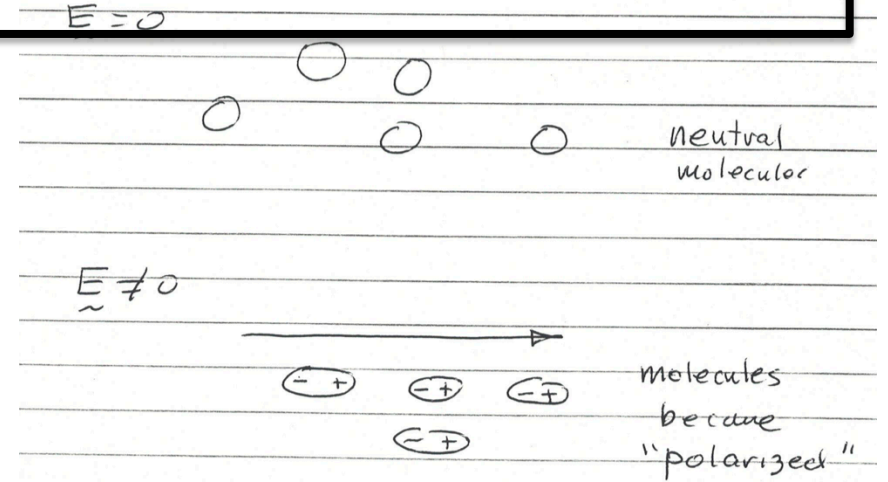
Dielectric Terminology

$$\nabla \times \frac{\mathbf{B}}{\mu_0} = \mathbf{J}_f + \frac{\partial}{\partial t}\left[\varepsilon_0\left(1 + \chi_E\right)\mathbf{E}\right]$$

$$\nabla \times \frac{\mathbf{B}}{\mu_0} = \mathbf{J}_f + \frac{\partial}{\partial t}\mathbf{D}$$

$$\mathbf{D} = \varepsilon_0\left(1 + \chi_E\right)\mathbf{E} = \varepsilon\mathbf{E}$$

$\mathbf{D}$ Electric flux density

$\varepsilon = \varepsilon_0\left(1 + \chi_E\right)$ Dielectric contant

$\left(1 + \chi_E\right)$ Relative Dielectric constant

Maxwell's Equations in Matter

Basic Equations (Vacuum)

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = Q/\varepsilon_0$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\int_S d\vec{\mathbf{A}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\oint_{Loop} \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{l}} = \mu_0 \int_S d\vec{\mathbf{A}} \cdot \left[\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right]$$

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \left[\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \right]$$

Here ρ and J are the <u>total charge and current densities</u>

Includes charge and current densities induced in dielectric and magnetic materials

Separate charge and current densities into "free" and "induced" components

Somewhat arbitrary but very useful

magnetization current

$$\mathbf{J} = \mathbf{J}_f + \mathbf{J}_m + \mathbf{J}_p$$

polarization current

"Free" current

polarization charge density

$$\rho = \rho_f + \rho_p$$

polarization density

"Free" charge density

$$\rho_p = -\nabla \cdot \varepsilon_0 \chi \mathbf{E} = -\nabla \cdot \mathbf{P}$$

$E = 0$

neutral molecular

$$\mathbf{J}_m = \nabla \times \mathbf{M}$$

$$\mathbf{J}_p = \varepsilon_0 \chi \frac{\partial \mathbf{E}}{\partial t}$$

magnetization density

$E \neq 0$

molecules become "polarized"
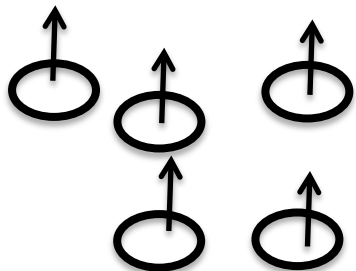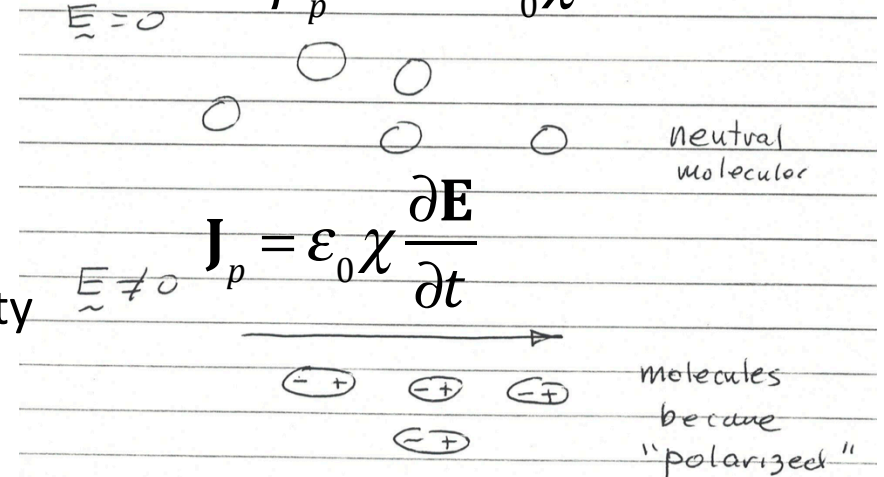
$$\mathbf{M} = \mu_0 \chi_m \mathbf{H}$$

Maxwell's Equations in Matter

Equations in linear media

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E}$$

$$\mathbf{P} = \varepsilon_0 \chi_E \mathbf{E}$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} = \mu \mathbf{H}$$

$$\mathbf{M} = \mu_0 \chi_M \mathbf{H}$$

$$\oint \vec{\mathbf{D}} \cdot d\vec{\mathbf{A}} = Q_{free}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\int_S d\vec{\mathbf{A}} \cdot \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\oint_{Loop} \vec{\mathbf{H}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{l}} = \int_S d\vec{\mathbf{A}} \cdot \left[\vec{\mathbf{J}}_{free} + \frac{\partial \vec{\mathbf{D}}}{\partial t} \right]$$

$$\nabla \cdot \vec{\mathbf{D}} = \rho_{free}$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}}_{free} + \frac{\partial \vec{\mathbf{D}}}{\partial t}$$

Maxwell's Equations in Matter

$$\nabla \cdot \mathbf{D} = \rho_f \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$$

(linear and isotropic matter)

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} = \frac{1}{\mu}\mathbf{B} , \quad \mathbf{J}_b = \nabla \times \mathbf{M}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon \mathbf{E} , \qquad \rho_b = -\nabla \cdot \mathbf{P}$$

$\mathbf{M} = \chi_m \mathbf{H}$: Magnetization field

$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$: Polarzation field

$\mu = \mu_0 (1 + \chi_m)$: permeability

$\varepsilon = \varepsilon_0 (1 + \chi_e)$:permittivity

$\mu_r = 1 + \chi_m$: relative permeability

$\varepsilon_r = 1 + \chi_e$: relative permittiity

χ_m : magnetic susceptibility

χ_e : electric suscepibility

In a good conductor – Ohms' Law (point version)

$$\mathbf{J}_f = \sigma\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

Boundary Conditions

Medium #1

Medium #2

$\varepsilon_1, \mu_1, \sigma_1$

$\varepsilon_2, \mu_2, \sigma_2$

How are components
of the fields on one
side of the boundary
related to those on
the other side?

$\mathbf{E}_1$

$\mathbf{E}_2$

$\mathbf{D}_1$

$\mathbf{D}_2$

$\mathbf{B}_1$

$\mathbf{B}_2$

$\mathbf{H}_1$

$\mathbf{H}_1$

General Comments

Tangential components of **E** are always equal.

Normal components of **B** are always equal.

Normal component of **E** discontinuous implies a **surface charge density.**

Normal components of **D** discontinuous implies a **free surface charge density**

Tangential components of **B** discontinuous implies a **surface current density.**

Tangential components of **H** discontinuous implies a **free surface current density.**

Induced surface charge density
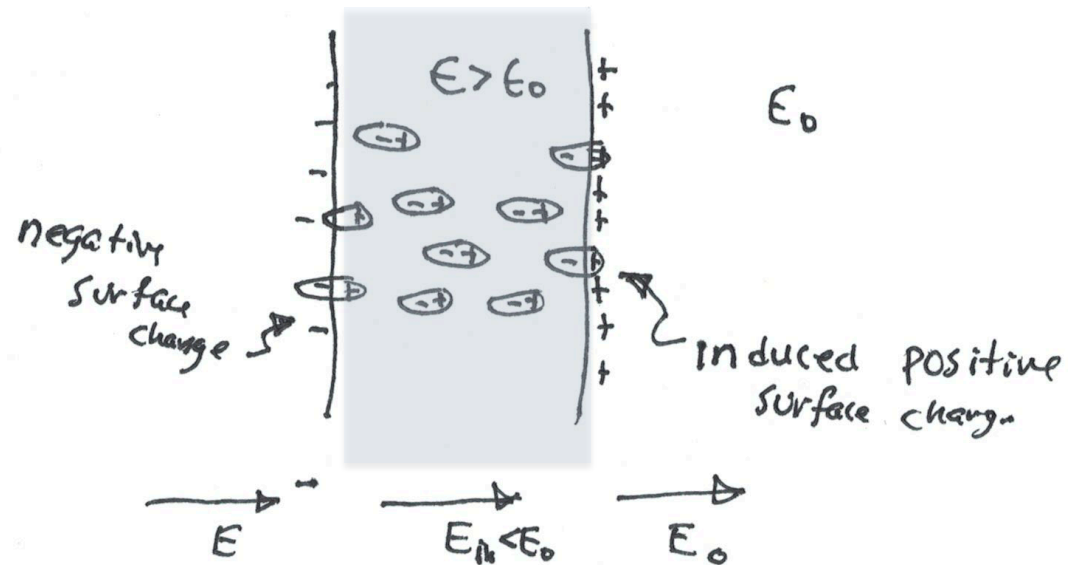
Dielectric Sphere
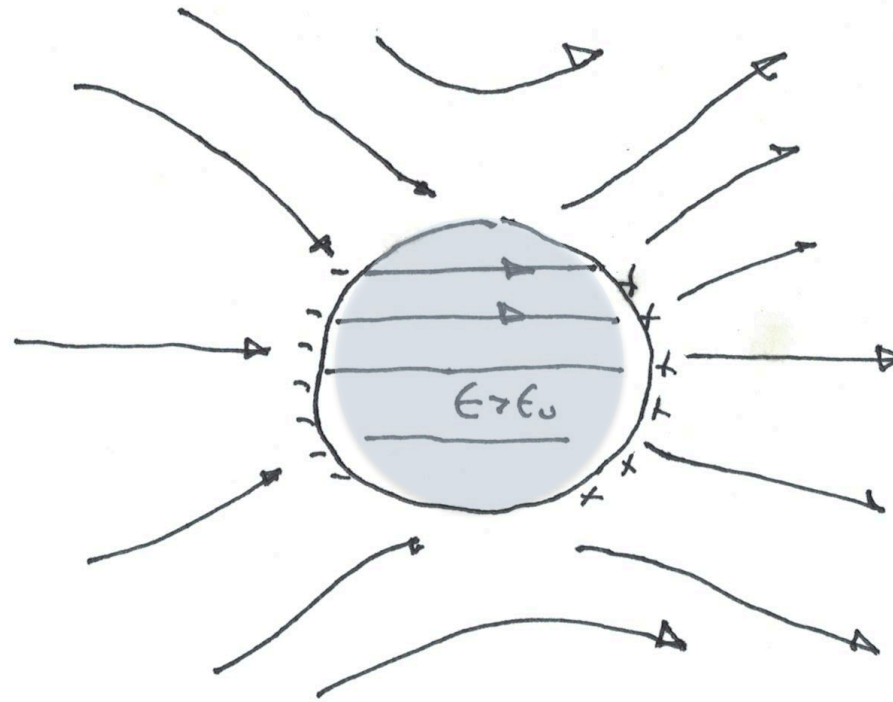
Magnetized Rod
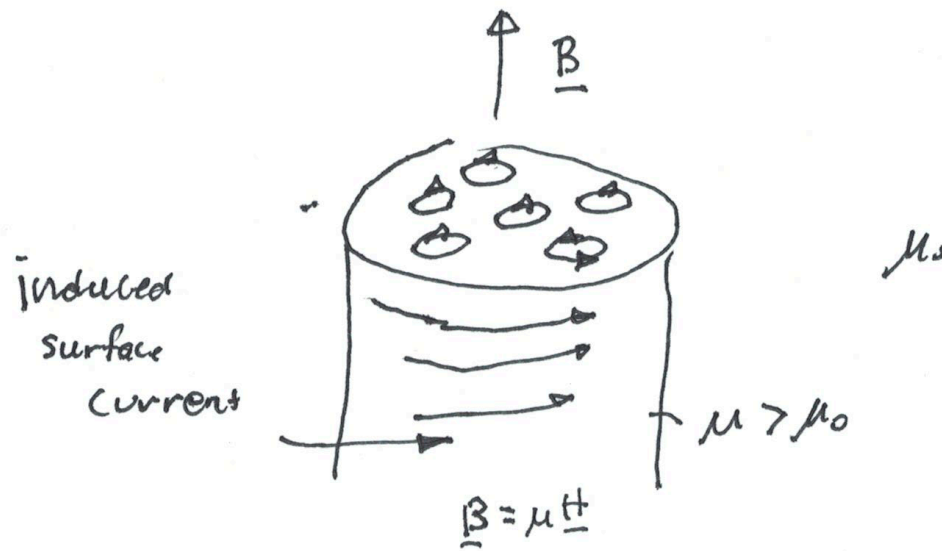
Conducting Sphere
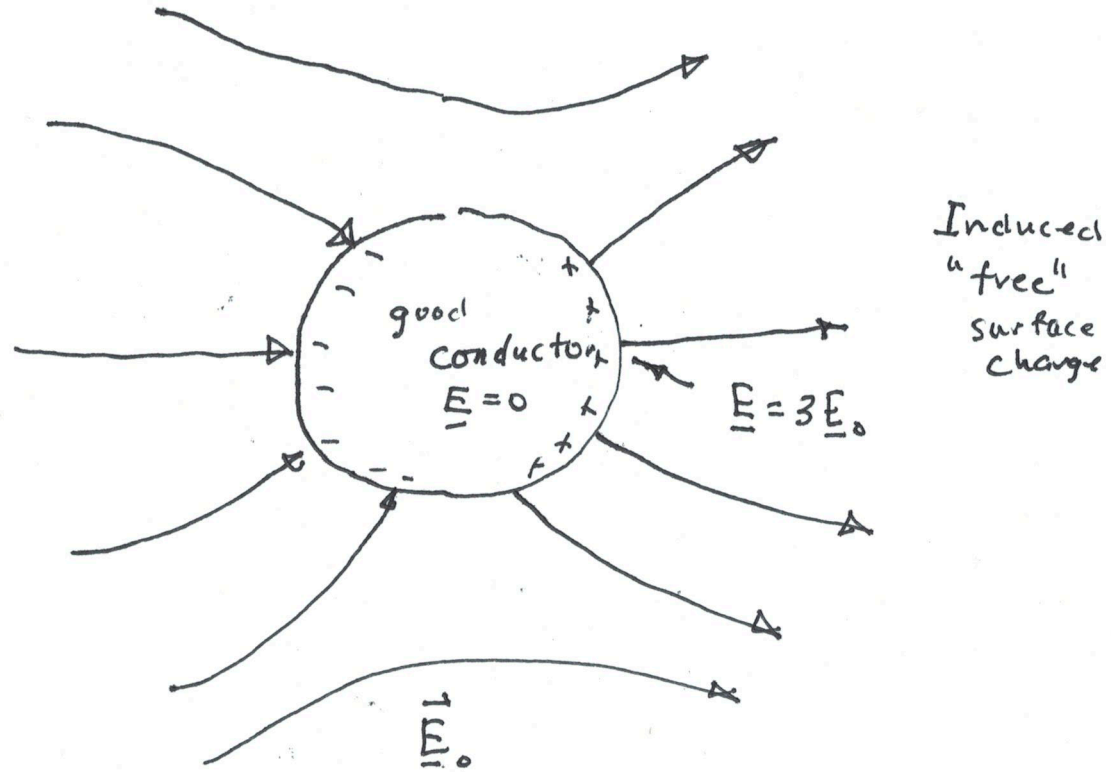
Tangential Component of **E**
Boundary Conditions

Applying Stokes's theorem

$$\int_A (\nabla \times \mathbf{E}) \cdot \mathbf{da} = -\int_A \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{da} = \oint_C \mathbf{E} \cdot \mathbf{dl}$$

boundary

$\mathbf{E}_{T2}$

Δl

$E_{T2}\Delta l + E_{T1}(-\Delta l) = 0$

$h \to 0$

$\int_A \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{da} \to 0$ since $h \to 0$

C:contour

$\mathbf{E}_{T1}$

$\to \boxed{E_{T2} = E_{T1}}$

- **The tangential component of electric field E is continuous across a boundary**

Normal Component of $\mathbf{D}$

Consider the electric flux field: $\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0(1+\chi_e)\mathbf{E}$ (linear/isotropic matter)

$\nabla \cdot \mathbf{D} = \rho_f \qquad \rho_f = $ free charge density

Applying the Divergence theorem $\displaystyle\int_A \mathbf{D} \cdot \hat{\mathbf{n}}\,da = \int_V \rho_f(\mathbf{r})d^3r$

$$\rho_f d^3r = \sigma_f da$$

ε_2

$\hat{\mathbf{n}}$ $\mathbf{D}_{N2}$

σ_f $\qquad h \to 0 \qquad \sigma_f :$ free surface

charge density

ε_1 $\hat{\mathbf{n}}$ $\mathbf{D}_{N1}$

$\Delta a :$ area
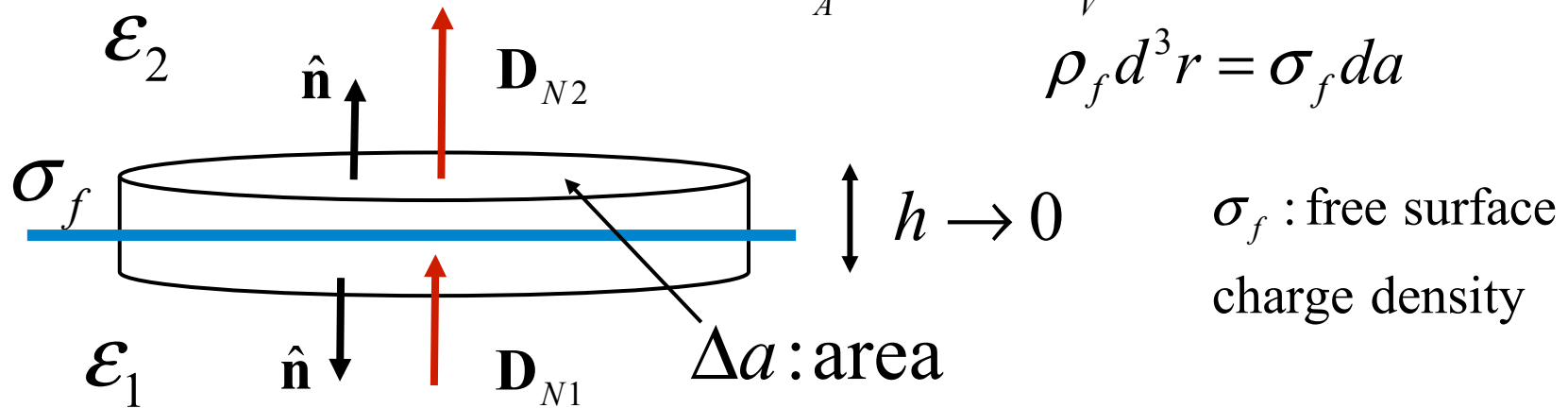
$$(\mathbf{D}_{N2} - \mathbf{D}_{N1}) \cdot \hat{\mathbf{n}}\Delta a = Q_f = \sigma_f \Delta a \qquad \rightarrow \qquad \boxed{\varepsilon_2 E_{N2} - \varepsilon_1 E_{N1} = \sigma_f}$$

- The **normal** component of the **electric flux field D** is discontinuous by the **free surface charge density**

Normal E?

$$\nabla \cdot \varepsilon_0 \mathbf{E} = \rho_t \qquad \rho_t = \text{total charge density}$$

Applying the Divergence theorem $\displaystyle\int_A \varepsilon_0 \mathbf{E} \cdot \hat{\mathbf{n}} \, da = \int_V \rho_t(\mathbf{r}) d^3 r$

$$\rho_t d^3 r = \sigma_t \, da$$

ε_2 $\hat{\mathbf{n}}$ $\mathbf{E}_{N2}$

σ_t $h \to 0$ σ_t : total surface

charge density

ε_1 $\hat{\mathbf{n}}$ $\mathbf{E}_{N1}$ Δa : area

$$\varepsilon_0 (\mathbf{E}_{N2} - \mathbf{E}_{N1}) \cdot \hat{\mathbf{n}} \Delta a = Q_t = \sigma_t \Delta a \quad \to \quad \boxed{\varepsilon_0 E_{N2} - \varepsilon_0 E_{N1} = \sigma_t}$$

- The **normal** component of the **electric field E** is discontinuous by the **total surface charge density**/ ε_0

Tangential Component of **H**

Magnetic field: $\nabla \times \mathbf{H} = \mathbf{J}_f + \dfrac{\partial \mathbf{D}}{\partial t}$

$\mathbf{D} = \varepsilon \mathbf{E}$ (linear/isotropic matter)

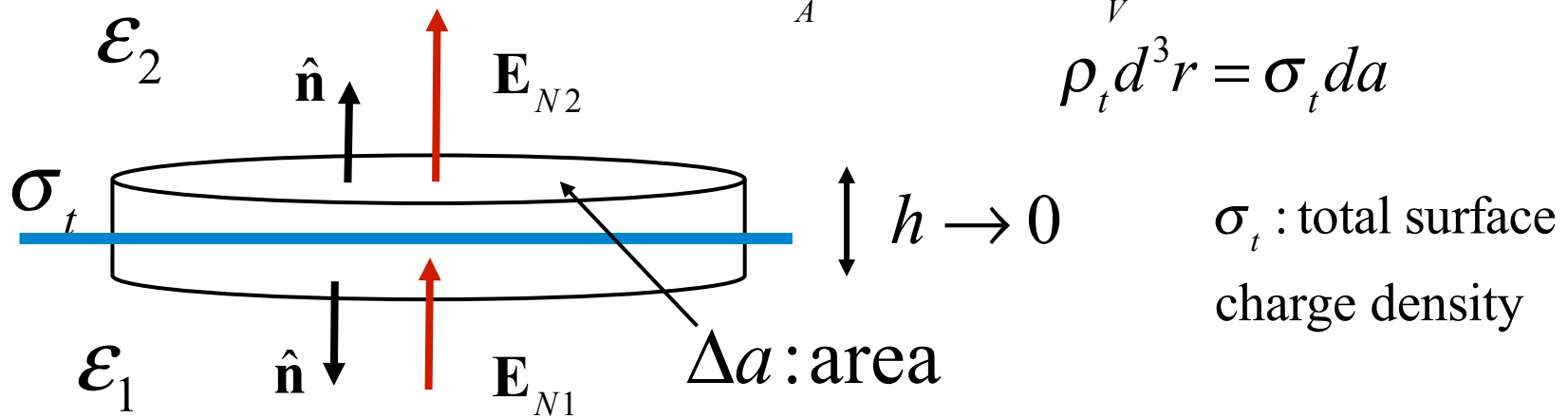$\mathbf{H} = \mathbf{B}/\mu$ (linear/isotropic matter and nonferromagnetic)

Applying Stokes's theorem

$$\oint_C \mathbf{H} \cdot \mathbf{dl} = \int_A \mathbf{J}_f \cdot \mathbf{da} + \int_A \dfrac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{da}$$

$$\int_A \mathbf{J}_f \cdot \mathbf{da} \;\rightarrow\; \mathbf{J}_f \Delta l h \;\rightarrow\; \mathbf{K}_f \Delta l$$

as $h \rightarrow 0$

C : contour

$\hat{\mathbf{n}}$

$\mathbf{H}_{T2}$

Δl

boundary

$h \rightarrow 0$ $\int_A \dfrac{\partial \mathbf{D}}{\partial t} \cdot \mathbf{da} \rightarrow 0$ since $h \rightarrow 0$

$\mathbf{H}_{T1}$

$\mathbf{K}_f$ $\hat{\mathbf{n}}$

free surface

current density

$$\mathbf{H}_{T2}\Delta l + \mathbf{H}_{T1}(-\Delta l) = \Delta l \mathbf{K}_f \times \hat{\mathbf{n}}$$

$$\boxed{\mathbf{H}_{T2} - \mathbf{H}_{T1} = \mathbf{K}_f \times \hat{\mathbf{n}}}$$

$\hat{\mathbf{n}}$: outward normal to the surface boundary

- **The tangential** component of **magnetic field H is discontinuous by the free surface current**

Tangential B

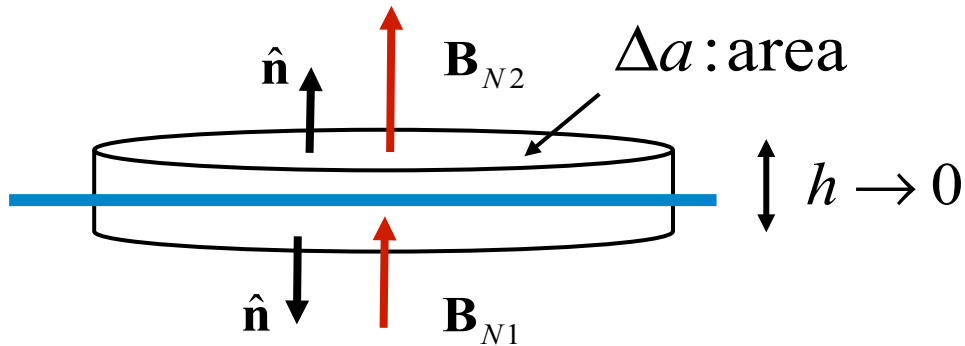$$\mathbf{H}_{T2} - \mathbf{H}_{T1} = \mathbf{K}_f \times \hat{\mathbf{n}}$$

$$\mathbf{B}_{T2} - \mathbf{B}_{T1} = \mu_0 \mathbf{K}_t \times \hat{\mathbf{n}}$$

- **The tangential** component of **magnetic flux density B is discontinuous by the total surface current** $\times \mu_0$

Normal Component of **B**

$$\nabla \cdot \mathbf{B} = 0 \qquad \text{Magnetic flux density} : \mathbf{B}$$

$$\oint_A \mathbf{B} \cdot \hat{\mathbf{n}} \, da = 0$$

$\hat{\mathbf{n}}$ $\mathbf{B}_{N2}$ $\Delta a : \text{area}$ $h \to 0$ $\hat{\mathbf{n}}$ $\mathbf{B}_{N1}$

$$(\mathbf{B}_{N2} - \mathbf{B}_{N1}) \cdot \hat{\mathbf{n}} \Delta a = 0 \quad \rightarrow \quad \boxed{\mathbf{B}_{N2} = \mathbf{B}_{N1}}$$

Normal compontent of **H** $\qquad \nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}$

$$\rightarrow \quad \boxed{H_{N2} - H_{N1} = -(M_{N2} - M_{N1})}$$

- The **normal** component of the **magnetic flux field B** is **continuous** across boundary

Boundary Condition Summary

Tangential Components

$$\mathbf{E}_{T2} = \mathbf{E}_{T1} \qquad \mathbf{H}_{T2} - \mathbf{H}_{T1} = \mathbf{K}_f \times \hat{\mathbf{n}} \qquad \mathbf{B}_{T2} - \mathbf{B}_{T1} = \mu_0 \mathbf{K}_t \times \hat{\mathbf{n}}$$

Normal Components

$$(\mathbf{D}_{N2} - \mathbf{D}_{N1}) \cdot \hat{\mathbf{n}} \Delta a = Q_f = \sigma_f \Delta a \quad \rightarrow \quad \boxed{\varepsilon_2 E_{N2} - \varepsilon_1 E_{N1} = \sigma_f}$$

$$\varepsilon_0 (\mathbf{E}_{N2} - \mathbf{E}_{N1}) \cdot \hat{\mathbf{n}} \Delta a = Q_t = \sigma_t \Delta a \quad \rightarrow \quad \boxed{\varepsilon_0 E_{N2} - \varepsilon_0 E_{N1} = \sigma_t}$$

$$(\mathbf{B}_{N2} - \mathbf{B}_{N1}) \cdot \hat{\mathbf{n}} \Delta a = 0 \quad \rightarrow \quad \boxed{\mathbf{B}_{N2} = \mathbf{B}_{N1}}$$

$$\boxed{H_{N2} - H_{N1} = -(M_{N2} - M_{N1})}$$

Let's play the boundary condition game!
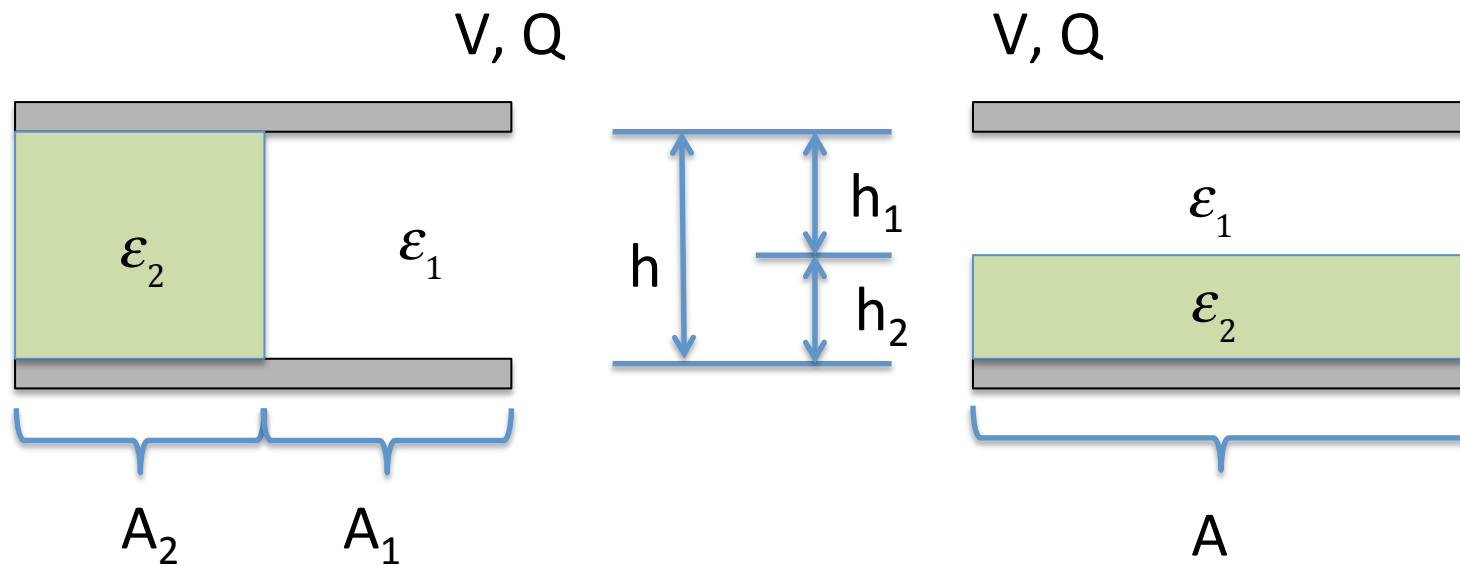
No free surface charge of current

x

z

y

Medium #1

$$\varepsilon_1 = 2\varepsilon_0, \mu_1 = \mu_0, \sigma_1 = 0$$

$$\mathbf{E}_1 = 2\hat{\mathbf{x}} + 4\hat{\mathbf{y}} + 6\hat{\mathbf{z}}$$

$$\mathbf{D}_1/\varepsilon_0 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$$

$$\mathbf{B}_1/\mu_0 = 6\hat{\mathbf{x}} + 7\hat{\mathbf{y}} + 8\hat{\mathbf{z}}$$

$$\mathbf{H}_1 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$$

Medium #2

$$\varepsilon_2 = 4\varepsilon_0, \mu_2 = 2\mu_0, \sigma_2 = 0$$

$$\mathbf{E}_2 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$$

$$\mathbf{D}_2/\varepsilon_0 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$$

$$\mathbf{B}_2/\mu_0 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$$

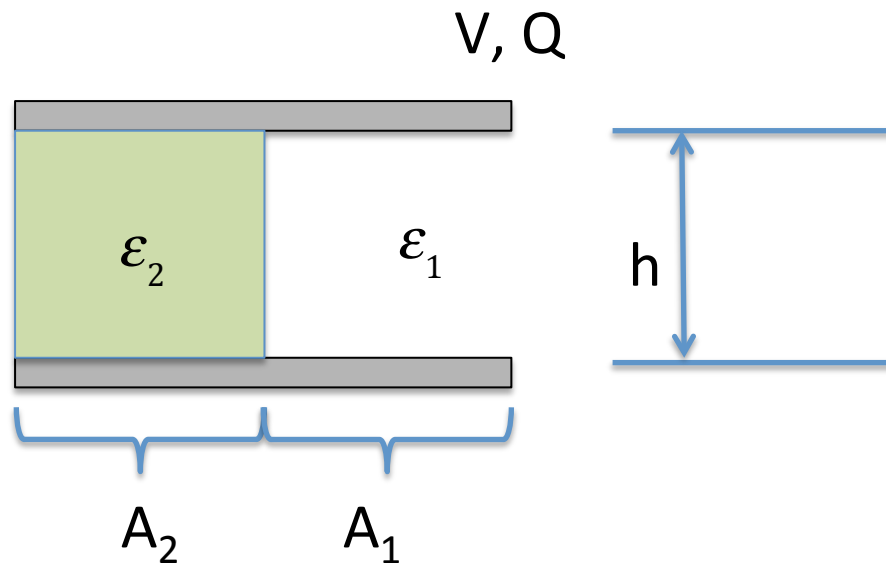$$\mathbf{H}_2 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$$

Boundary Conditions in a Capacitor

V, Q

V, Q

ε_2

ε_1

h_1

h

h_2

ε_1

ε_2

A_2

A_1

A

Which boundary Conditions Apply?

Boundary Conditions in a Capacitor

V, Q

ε_2 ε_1 h

A_2 A_1

Case #1

$$E_1 = E_2 = V/h \quad \text{tangential E}$$

$$Q_1 = A_1 D_1 = A_1 \varepsilon_1 V / h$$

$$Q_2 = A_2 D_2 = A_2 \varepsilon_2 V / h$$

Case #1

$$Q = Q_1 + Q_2 = \left(\frac{A_1 \varepsilon_1 + A_2 \varepsilon_2}{h} \right) V$$

$$C = \left(\frac{A_1 \varepsilon_1 + A_2 \varepsilon_2}{h} \right) \text{ Capacitors in parallel}$$

Boundary Conditions in a Capacitor

V, Q

h_1 ε_1

h_2 ε_2

A
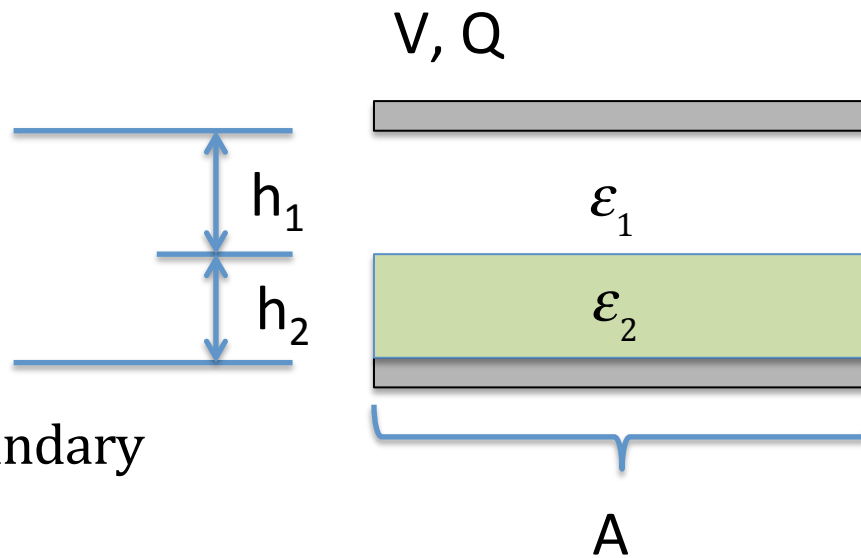
Case #2

No free surface charge on boundary

between ε_1 and ε_2.

$$D_1 = D_2 = Q/A$$

$$E_1 = D_1/\varepsilon_1 = Q/(\varepsilon_1 A)$$

$$E_2 = D_2/\varepsilon_2 = Q/(\varepsilon_2 A)$$

Case #2

$$V = h_1 E_1 + h_2 E_2 = Q\left(\frac{h_1}{A\varepsilon_1} + \frac{h_2}{A\varepsilon_2}\right)$$

$$C^{-1} = \left(\frac{h_1}{A\varepsilon_1} + \frac{h_2}{A\varepsilon_2}\right) \quad \text{Capacitors in series}$$

Let's play the boundary condition game! With conductivity!!

x

z

y

Medium #1

$\varepsilon_1 = 2\varepsilon_0, \mu_1 = \mu_0, \sigma_1 = \sigma$

$\mathbf{E}_1 = 2\hat{\mathbf{x}} + 4\hat{\mathbf{y}} + 6\hat{\mathbf{z}}$

$\mathbf{D}_1 / \varepsilon_0 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$

$\mathbf{B}_1 / \mu_0 = 6\hat{\mathbf{x}} + 7\hat{\mathbf{y}} + 8\hat{\mathbf{z}}$

$\mathbf{H}_1 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$

$\mathbf{J}_1 / \sigma = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$

Medium #2

$\varepsilon_2 = 4\varepsilon_0, \mu_2 = \mu_0, \sigma_2 = 2\sigma$

$\mathbf{E}_2 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$

$\mathbf{D}_2 / \varepsilon_0 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$

$\mathbf{B}_2 / \mu_0 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$

$\mathbf{H}_2 = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$

$\mathbf{J}_2 / \sigma = \hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}$