

ENEE 222: 4/04 Class

Material: Lecture videos 15.2, 16.1, 16.2

1. Given that

$$[1 \quad 2 \quad 4 \quad 8]^T \xleftrightarrow{\text{DFT}} [15 \quad -3 + 6j \quad -5 \quad -3 - 6j]^T,$$

which of the following vectors is the DFT of

$$[15 \quad -6 - 3j \quad 5 \quad -6 + 3j]^T ?$$

- A. $[4 \quad 2 \quad 16 \quad 32]^T$
- B. $[4 \quad 32 \quad 16 \quad 8]^T$
- C. $[8 \quad 4 \quad 32 \quad 16]^T$
- D. $[16 \quad 8 \quad 4 \quad 32]^T$

2. The circular convolution of

$$[1 \quad 2 \quad 4 \quad 8]^T \quad \text{and} \quad [a \quad b \quad c \quad d]^T$$

produces a vector whose last entry equals

- A. $a + 2b + 4c + 8d$
- B. $a + 8b + 4c + 2d$
- C. $2a + 4b + 8c + d$
- D. $8a + 4b + 2c + d$

3. Let \mathbf{x} be a real-valued vector of length $N > 2$ and \mathbf{y} be a Fourier sinusoid for that length (i.e., $\mathbf{y} = \mathbf{v}^{(m)}$ for some integer m). Which (one or more) the following statements is true about the circular convolution vector

$$\mathbf{s} = \mathbf{x} \circledast \mathbf{y} ?$$

- A. \mathbf{s} is real-valued.
- B. \mathbf{s} is circularly conjugate-symmetric.
- C. The DFT \mathbf{S} contains only one nonzero entry.
- D. \mathbf{s} is a scalar multiple of \mathbf{y} .

4. Suppose the DFT of the vector

$$[a \quad b \quad c \quad d]^T$$

contains no zero entries. Which (one or more) of the following vectors \mathbf{x} is certain to have *exactly* four nonzero entries in its DFT \mathbf{X} ?

- A. $\mathbf{x} = [a \quad b \quad c \quad d \quad a \quad b]^T$
- B. $\mathbf{x} = [a \quad b \quad c \quad d \quad a \quad b \quad c \quad d]^T$
- C. $\mathbf{x} = [a \quad b \quad c \quad d \quad a \quad b \quad c \quad d \quad a \quad b]^T$
- D. $\mathbf{x} = [a \quad b \quad c \quad d \quad a \quad b \quad c \quad d \quad a \quad b \quad c \quad d \quad a \quad b \quad c \quad d]^T$

5. (HW 18 ~v) Let $N > 2$. For what values of ω_0 in $(0, \pi]$ is the signal vector \mathbf{u} defined by

$$u[n] = \cos(\omega_0 n), \quad n = 0 : N - 1$$

circularly symmetric?

6. (HW 19 ~ii) How are $\mathbf{x} \otimes \mathbf{y}$ and $(\mathbf{P}\mathbf{x}) \otimes \mathbf{y}$ related?

7. (HW 19 iii) If

$$\begin{bmatrix} 2 & -2 & 3 & -3 & 5 & -1 \\ -1 & 2 & -2 & 3 & -3 & 5 \\ 5 & -1 & 2 & -2 & 3 & -3 \\ -3 & 5 & -1 & 2 & -2 & 3 \\ 3 & -3 & 5 & -1 & 2 & -2 \\ -2 & 3 & -3 & 5 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \\ 9 \\ 2 \\ 12 \\ -7 \end{bmatrix},$$

explain how the vector \mathbf{x} can be obtained using DFTs (as opposed to a conventional solution of a 6×6 system of linear equations).