ENEE 222: 3/12 Class

Material: Lecture videos **11.1**, **11.2**

1. If

$$\mathbf{u} = \begin{bmatrix} 5 & j & 1-j \end{bmatrix}^T \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 1 & 2+j & 3 \end{bmatrix}^T ,$$

then the inner product $\langle \mathbf{u}, \mathbf{v} \rangle$ equals

A. $7+j$

B. $7-j$

C. $9+j$

D. $9-j$

2. Vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are mutually orthogonal, each having norm equal to 2. If

$$\begin{aligned} \mathbf{x} &= \mathbf{u} + 2j\mathbf{v} - 3\mathbf{w} \\ \mathbf{y} &= (1+j)\mathbf{u} - \mathbf{v} + \mathbf{w} , \end{aligned}$$

then $\langle \mathbf{x}, \mathbf{y} \rangle$ equals

A. $-2+3j$

B. $-2-j$

C. $-8+12j$

D. $-8-4j$

3. If

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} ,$$

then $4 \cdot c_4$ equals

A. $s_1 + s_2 + s_3 + s_4$

B. $s_1 + js_2 - s_3 - js_4$

C. $s_1 - s_2 + s_3 - s_4$

D. $s_1 - js_2 - s_3 + js_4$

4. (HW 12 i) Determine the real values a, b and $c > 0$ such that

$$\mathbf{V} = \left[\begin{array}{cccc} \mathbf{v}^{(1)} & \mathbf{v}^{(2)} & \mathbf{v}^{(3)} & \mathbf{v}^{(4)} \end{array}\right] = \left[\begin{array}{cccc} 2 & 3+jc & -8 & a+jb \\ a+jb & 2 & 3+jc & -8 \\ -8 & a+jb & 2 & 3+jc \\ 3+jc & -8 & a+jb & 2 \end{array}\right]$$

has orthogonal columns. *Assume these values in what follows.*

5. (HW 12 iv) If

$$\begin{aligned} \mathbf{x} &= \mathbf{v}^{(1)} + 2j\mathbf{v}^{(2)} + \mathbf{v}^{(3)} + 2j\mathbf{v}^{(4)} \\ \mathbf{y} &= j\mathbf{v}^{(1)} - 3\mathbf{v}^{(2)} - 3\mathbf{v}^{(3)} + j\mathbf{v}^{(4)} \end{aligned}$$

determine $\|\mathbf{x} - \mathbf{y}\|^2$ *without using* any of the numerical entries of the vectors $\mathbf{v}^{(i)}$.

6. (HW 12 $\subset$ ii, iii) Determine the projection $\hat{\mathbf{s}}$ of

$$\mathbf{s} = \left[\begin{array}{cccc} 27 & 45 & 41 & 23 \end{array}\right]^T$$

onto $\mathbf{v}^{(2)}$. What is the resulting squared error norm $\|\mathbf{s} - \hat{\mathbf{s}}\|^2$?