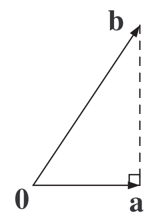
ENEE 222: 3/07 Class

Material: Lecture videos **10.1**, **10.2**

1. If vectors $\mathbf{a}$ and $\mathbf{b}$ are as shown in the figure below, which (one or more) of the following statements are true?



A. Vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.

B. Vectors $\mathbf{a}$ and $\mathbf{b} - \mathbf{a}$ are orthogonal.

C. $\langle \mathbf{a}, \mathbf{b} \rangle = \|\mathbf{a}\| \cdot \|\mathbf{b}\|$

D. $\langle \mathbf{a}, \mathbf{b} \rangle = \|\mathbf{a}\|^2$

2. The angle between $\mathbf{a} = [3 \quad 1 \quad 1 \quad 5]^T$ and $\mathbf{b} = [1 \quad -5 \quad -1 \quad -3]^T$ equals

A. $\pi/3$

B. $\cos^{-1}(-1/8)$

C. $\cos^{-1}(-1/72)$

D. $2\pi/3$

3. Vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are mutually orthogonal and such that $\|\mathbf{u}\| = 1$, $\|\mathbf{v}\| = 2$ and $\|\mathbf{w}\| = 3$. Let

$$\mathbf{s} = 5\mathbf{u} + 7\mathbf{v} - 2\mathbf{w}$$

If $\hat{\mathbf{s}}$ is the projection of $\mathbf{s}$ onto the subspace defined by $\mathbf{u}$ and $\mathbf{v}$, then $\|\mathbf{s} - \hat{\mathbf{s}}\|$ equals

A. 6

B. 12

C. 18

D. -6

4. (HW 11 i) Show that

$$\mathbf{v}^{(1)} = [3 \quad 1 \quad 1 \quad -1]^T, \qquad \mathbf{v}^{(2)} = [1 \quad -2 \quad 0 \quad 1]^T \qquad \text{and} \qquad \mathbf{v}^{(3)} = [-1 \quad 1 \quad 5 \quad 3]^T$$

are mutually orthogonal, and compute their norms.

5. (HW 11 ii) If $\mathbf{s} = [1 \quad 6 \quad 2 \quad 7]^T$, determine the projection $\mathbf{f}^{(i)}$ of $\mathbf{s}$ onto each $\mathbf{v}^{(i)}$.

6. (HW 11 iv) Determine the angle between $\mathbf{s}$ and the plane defined by $\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}$.

7. (HW 11 v, vi) If $\mathbf{g}$ is the projection of $\mathbf{s}$ onto the three-dimensional subspace defined by $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}$, determine the error vector $\mathbf{s} - \mathbf{g}$. What special property does $\mathbf{s} - \mathbf{g}$ have?

8. (HW 11 $\supset$ vii) Solve the system

$$\begin{bmatrix} 3 & 1 & -1 & 1 \\ 1 & -2 & 1 & 2 \\ 1 & 0 & 5 & -2 \\ -1 & 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

without using Gaussian elimination.