ENEE 222: 2/26 Class

Material: Lecture videos **9.1**, **9.2**, **9.3**

1. Which of the following linear transformations $\mathbf{R}^3 \to \mathbf{R}^3$ are invertible?

A. Rotation of a vector $\mathbf{x}$ about the vertical (x_3) axis through angle θ.

B. Reflection of a vector $\mathbf{x}$ across the plane $x_1 = x_2$.

C. Projection of a vector $\mathbf{x}$ onto the plane $x_1 = x_2$.

D. Reflection of a vector $\mathbf{x}$ across the horizontal (x_1) axis.

2. If

$$\mathbf{A}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{A}\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

then $\mathbf{A}^{-1} =$

A. $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ B. $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ C. $\begin{bmatrix} -1 & 1 \\ 4 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 2 & -3 \\ 3 & 1 \end{bmatrix}$

3. You are given

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{L}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ a & b & c & d \end{bmatrix}$$

What is the value of a?

A. 0 B. 3 C. 4 D. -3

4. Here's a section of a Gaussian elimination table:

m	x_1	x_2	x_3	b
	4	5	2	12
-1	4	7	-2	4
c	3	3	2	9
	4	5	2	12
	0	2	-4	-8
3/8	0	$-3/4$	1/2	0

What is the value of the multiplier c?

A. 3/4 B. $-3/4$ C. 4/3 D. $-4/3$

5. (HW 10 i) Let

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1/2 & -3/2 & 2 & 0 \\ 1 & 4 & 1 & 1 \end{bmatrix}$$

Solve $\mathbf{Lx} = \mathbf{b}$ for an arbitrary vector $\mathbf{b}$. Display $\mathbf{L}^{-1}$.

6. (HW 10 $\subset$ ii) Let $\mathbf{G} = \mathbf{PBD}$, where $\mathbf{P}$ is a permutation, $\mathbf{B}$ is invertible and $\mathbf{D}$ is diagonal (and invertible also). Write an expression for $\mathbf{G}^{-1}$, and interpret the operations in that expression in terms of scaling and permutation of rows/columns.

7. (HW 10 $\subset$ iii) If

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 & 1 \\ 2 & -5 & 1 & 7 \\ 1 & 5 & 2 & -2 \\ 2 & -11 & 5 & 13 \end{bmatrix}$$

and $\mathbf{b} = \begin{bmatrix} 12 & 8 & 1 & 6c \end{bmatrix}^T$, reduce the equation $\mathbf{Ax} = \mathbf{b}$ to $\mathbf{Ux} = \mathbf{r}$, where $\mathbf{U}$ is upper-triangular.