ENEE 222 : 02/21 Class

Material: Lecture videos **8.1**, **8.2**

1. If

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 4 \\ 2 & 0 & 3 \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{bmatrix} 5 & -1 \\ 1 & 2 \\ -1 & 3 \end{bmatrix},$$

the product $\mathbf{AB}$ equals

A. $\begin{bmatrix} 1 & 7 \\ -7 & 7 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 7 \\ 7 & 7 \end{bmatrix}$ C. $\begin{bmatrix} 1 & -7 \\ 7 & 7 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 7 \\ 7 & -7 \end{bmatrix}$

2. Let $\mathbf{a}$ and $\mathbf{b}$ be n-dimensional column vectors (where $n > 1$) having real-valued entries. If

$$\mathbf{C} = \mathbf{a}^T\mathbf{b}\,\mathbf{b}^T\mathbf{a}\,,$$

which (one ore more) of the following statements are true about $\mathbf{C}$?

A. $\mathbf{C}$ is a $n \times n$ matrix

B. $\mathbf{C}$ is scalar (i.e., 1×1)

C. $\mathbf{C} = \mathbf{C}^T$

D. $\mathbf{C}$ may contain both positive and negative entries, depending on the choice of $\mathbf{a}$ and $\mathbf{b}$.

3. Let

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}, \qquad \mathbf{P} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \mathbf{AP}$$

Then $\mathbf{A} = \mathbf{BQ}$, where $\mathbf{Q} =$

A. $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

4. (HW 7 iii) Express the vector $\mathbf{v} = [1 \;\; 0 \;\; 0]^T$ as the sum of two vectors, one parallel to $\mathbf{s} = [1 \;\; 1 \;\; 1]^T$ and another perpendicular to $\mathbf{s}$. Hence determine the reflection of $\mathbf{v}$ about the plane through the origin which is normal to $\mathbf{s}$.

5. (HW 8 i) If $(r,\, s)$ is any point other than the origin on the Cartesian plane, determine the positive scaling factor α such that

$$\alpha(r,\, s) \;=\; (\cos\theta,\, \sin\theta)$$

for some (unique) angle θ. What geometric transformation does the matrix

$$\mathbf{A} \;=\; \begin{bmatrix} r & -s \\ s & r \end{bmatrix}$$

represent?

6. (HW 8 iii) If

$$\mathbf{A} \;=\; \begin{bmatrix} \cos(5\pi/24) & -\sin(5\pi/24) \\ \sin(5\pi/24) & \cos(5\pi/24) \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} \;=\; \begin{bmatrix} \cos(3\pi/16) & \sin(3\pi/16) \\ -\sin(3\pi/16) & \cos(3\pi/16) \end{bmatrix},$$

find the matrix $\mathbf{C}$ such that $\mathbf{A}^2\mathbf{C}\mathbf{B}^2$ equals the identity matrix $\mathbf{I}$.

7. ($\subset$HW 9) If

$$\mathbf{A} = \begin{bmatrix} a & b & 0 & c \\ d & e & f & 0 \\ 0 & r & s & t \\ u & 0 & v & w \end{bmatrix},$$

find matrices $\mathbf{P}$ and $\mathbf{Q}$ such that

$$\mathbf{PAQ} \;=\; \begin{bmatrix} a & b - r & -s \end{bmatrix}$$