ENEE 222: 2/19 Class

Material: Lecture videos **7.1**, **7.2**

1. Which of the following transformations $\mathbf{R}^2 \to \mathbf{R}^2$, described in geometric terms, does the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

represent?

A. Projection of a point on the horizontal (x_1) axis.

B. Projection of a point on the vertical (x_2) axis.

C. Counterclockwise rotation of a vector (from the origin to a point) by $\pi/2$ radians.

D. Reflection of a point about the straight line which bisects the first and third quadrants.

2. Which (one or more) of the following relationships between $\mathbf{x} = (x_1, x_2, x_3)$ and y represents a linear transformation $\mathbf{R}^3 \to \mathbf{R}$?

A. $y = x_1 + x_2 + x_3$

B.
$$y = \begin{cases} x_1, & x_1 \geq 0 \\ 0, & x_1 < 0 \end{cases}$$

C. y equals the length of the vector $\mathbf{x}$.

D. y equals the cosine of the angle between the vector $\mathbf{x}$ and the vector $(1, 1, 1)$.

3. If

$$\mathbf{A} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \mathbf{u} \quad \text{and} \quad \mathbf{A} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \mathbf{v}\ ,$$

then

$$\mathbf{A} \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$$

A. $\mathbf{u} + \mathbf{v}$

B. $\mathbf{u} - \mathbf{v}$

C. $2\mathbf{u} + 3\mathbf{v}$

D. $\mathbf{u} + 2\mathbf{v}$

4. If

$$\mathbf{A}\begin{bmatrix}1\\2\\0\end{bmatrix} = \begin{bmatrix}2\\1\\5\end{bmatrix} \quad \text{and} \quad \mathbf{A}\begin{bmatrix}-1\\1\\0\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix},$$

which (one or more) of the following statements are correct?

A. The dimensions of $\mathbf{A}$ cannot be determined based on the given information.

B. $\mathbf{A}$ is a 3×3 matrix.

C. The second column of $\mathbf{A}$ is given by $[\,1 \;\; 1 \;\; 2\,]^T$.

D. The entries of the third column of $\mathbf{A}$ cannot be determined based on the given information.

5. (HW 7 ⊂ii) Express each of the three standard unit vectors, $\mathbf{e}^{(1)}$, $\mathbf{e}^{(2)}$ and $\mathbf{e}^{(3)}$, as a linear combination of

$$\mathbf{x} = \begin{bmatrix}6\\0\\-3\end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix}3\\-3\\3\end{bmatrix} \quad \text{and} \quad \mathbf{z} = \begin{bmatrix}3\\3\\0\end{bmatrix}$$

6. (HW 7 iii) Express the vector $\mathbf{v} = [1 \;\; 0 \;\; 0]^T$ as the sum of two vectors, one parallel to $\mathbf{s} = [1 \;\; 1 \;\; 1]^T$ and another perpendicular to $\mathbf{s}$. Hence determine the reflection of $\mathbf{v}$ about the plane through the origin which is normal to $\mathbf{s}$.

7. (HW 8 i) If $(r,\, s)$ is any point other than the origin on the Cartesian plane, determine the positive scaling factor α such that

$$\alpha(r,\, s) = (\cos\theta,\, \sin\theta)$$

for some (unique) angle θ. What geometric transformation does the matrix

$$\mathbf{A} = \begin{bmatrix} r & -s \\ s & r \end{bmatrix}$$

represent?