Lecture 2, Part 3

Extensions of Complex Product

- n^{th} power of a complex number
- Inverse and complex conjugate
- Division of complex numbers

Computing z^n

$$z^n = (a + jb)^n$$

can be computed using the binomial theorem and $j^2 = -1$.

In polar coordinates,

$$|z^n| = |z|^n \qquad \text{and} \qquad \angle z^n = n(\angle z)$$

This is faster, but not necessarily more accurate.

Example. If $z = 1 + 2j$, the exact result for z^{10} is

$$(1 + 2j)^{10} = 237 - j(3116)$$

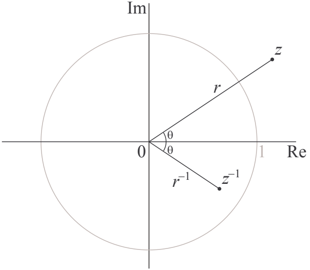
In polar coordinates,

$$|z| = \sqrt{5}, \qquad \angle z = \arctan(2) \approx 1.107$$
$$|z^{10}| = 3125, \qquad \angle z^{10} \approx 11.07$$

With the given precision, the result is (rather) unsatisfactory:

$$z^{10} \approx 235.4 - j(3116.1)$$

Inverse of a Complex Number



The inverse z^{-1} of $z \neq 0$ is defined by the identity

$$z^{-1}z = 1$$

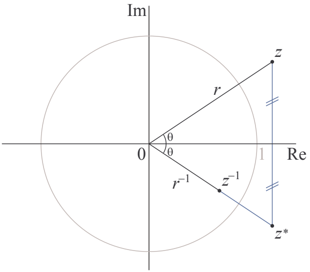
In polar coordinates, this is expressed as

$$|z^{-1}| \cdot |z| = 1 \qquad \text{and} \qquad \angle z^{-1} + \angle z = \angle 1 = 0 \ \text{(rad)}$$

In other words,

$$|z^{-1}| = |z|^{-1} \qquad \text{and} \qquad \angle z^{-1} = -\angle z$$

Complex Conjugate



The complex conjugate z^* is defined by

$$\Re e\{z^*\} = \Re e\{z\} \quad \text{and} \quad \Im m\{z^*\} = -\Im m\{z\}$$

Equivalently in polar coordinates,

$$|z^*| = |z| \quad \text{and} \quad \angle z^* = -\angle z$$

Note that z^* and z^{-1} are real multiples of each other, with

$$z^* = (|z|^2) \cdot z^{-1}$$

Complex Division

$$\frac{z_1}{z_2} \stackrel{\text{def}}{=} z_1(z_2)^{-1}$$

Therefore in polar coordinates,

$$|z_1/z_2| = |z_1|/|z_2| \quad \text{and} \quad \angle(z_1/z_2) = \angle z_1 - \angle z_2$$

In Cartesian coordinates,

$$\frac{a+jb}{c+jd} = \frac{a+jb}{c+jd} \cdot \frac{c-jd}{c-jd}$$

$$= \frac{(a+jb)(c-jd)}{c^2+d^2}$$

(In effect, $zz^* = |z|^2$ is used to invert the denominator.)

Complete by computing the product in the numerator.