Leave blank →

LAST NAME: **First Name:**

Grade:

ENEE 222 02* FALL 2018
FINAL EXAMINATION

- Closed book, no notes, no calculators.
- Show your work clearly, justifying your answers where appropriate.
- Enter your name (LAST followed by first) on all sheets.
- *Do not* detach any sheets. Use this sheet for rough work.
- Limit your answer to any single problem to both sides of the same sheet. If you need additional sheets, please inform the proctor. *Do not* continue on the opposite (left) page, or any other page.

	Periodic Time-Domain Signal	*Fourier Series Coefficients*
	$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t}$	$X_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\Omega_0 t}dt$
	$x^*(t)$	X_{-k}^*
	$x(-t)$	X_{-k}
	$x(t-D)$	$e^{-jk\Omega_0 D}X_k$
$(L \in \mathbf{Z})$	$x(t)e^{jL\Omega_0 t}$	X_{k-L}
$(\beta > 0)$	$x(\beta t)$	X_k

Common Trigonometric Values

$\cos(\pi/6) = \sin(\pi/3) = \sqrt{3}/2;$ $\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2;$ $\cos(\pi/3) = \sin(\pi/6) = 1/2$

Leave blank →

LAST NAME: **First Name:**

Grade:

PROBLEM 1 (15 pts.)

Let

$$x(t) \;=\; 2.1 \,+\, 3.7\cos(28\pi t - 1.4) \,+\, 5.3\cos(42\pi t + 0.5) \,+\, 1.9\cos(70\pi t - 2.3)\;,$$

where t is in seconds.

(i) (3 pts.) Determine the fundamental period T_0 of $x(t)$.

(ii) (5 pts.) Define (i.e., give the numerical values of) a scalar `c` and a vector `b` having the *smallest possible length*, such that

```
c =                               ;
b = [                       ].'   ;
z = c*ifft(b,300)                 ;
x = real(z)                       ;
```

generates a vector `x` consisting of 300 samples of $x(t)$ uniformly spaced in $[0, T_0)$.

(iii) (4 pts.) If `b` and `c` are as specified in **(ii)** above, the code

```
b_new = [1.0 ; conj(b)]         ;    % CONJ(.) : complex conjugate
z = c*ifft(b_new, 150)          ;
s = real(z)                     ;
```

produces 150 samples, uniformly spaced in $[0,\, T_0)$, of a periodic signal $s(t)$ with the same fundamental period (i.e., T_0) as $x(t)$. If $s(t)$ has no harmonics higher than the 74^{th}, write an equation for $s(t)$.

(iv) (3 pts.) Suppose that $x(t)$ is the input to an ideal *highpass* filter with unit gain, zero delay and cutoff frequency $f_c = 25$ Hz. Write an equation for the output signal $y(t)$.

. . . .

Leave blank →

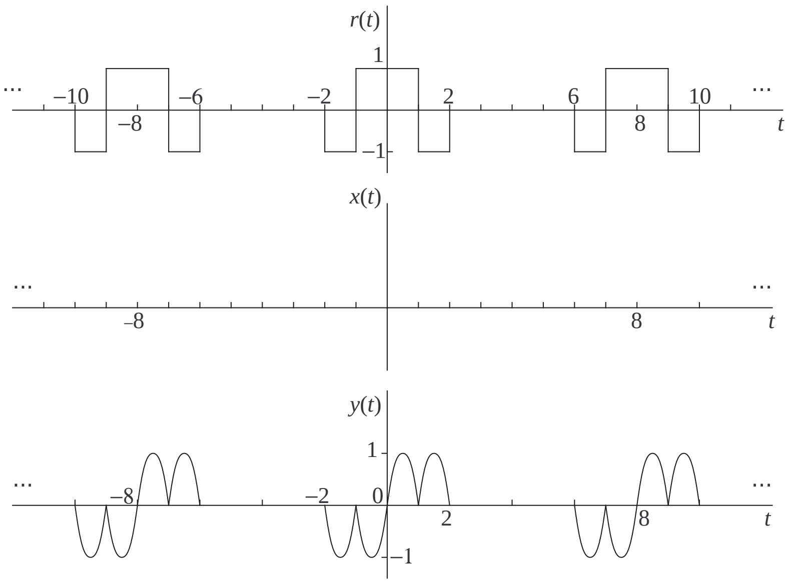
LAST NAME: **First Name:**

Grade:

PROBLEM 2 (15 pts.)

The signal $r(t)$ shown below is periodic with period $T_0 = 8$ and has complex Fourier series expansion

$$r(t) \;=\; \sum_{k=-\infty}^{\infty} R_k e^{jk\Omega_0 t} \;,$$



(i) (1 pt.) Determine the value of R_0.

(ii) (5 pts.) Write an equation (in terms of $r(t)$) for the real-valued signal $x(t)$ which is periodic with the same fundamental period as $r(t)$ and whose Fourier series coefficients are given by

$$X_0 \;=\; 1 \qquad \text{and} \qquad X_k \;=\; 2j \cdot R_k \sin(k\pi/2) \quad (k \neq 0)$$

(iii) (2 pts.) Sketch the signal $x(t)$ obtained in **(ii)** in the space provided (middle graph). Make sure to label the vertical axis.

(iv) (3 pts.) Express the real-valued periodic signal $y(t)$ (bottom graph) in terms of $r(t)$. *The curved sections of the graph are sinusoidal.*

(v) (4 pts.) Express each Y_k in terms of R_k's.

Continue on next sheet →

(Additional space for Problem 2 only)

Leave blank →

LAST NAME: **First Name:**

Grade:

PROBLEM 3 (15 pts.)

Consider the filter with impulse response

$$h[n] = 3\delta[n] - \delta[n-1] + 4\delta[n-2] - 4\delta[n-3] + \delta[n-4] - 3\delta[n-5]$$

(i) (5 pts.) Express the filter's frequency response $H(e^{j\omega})$ in the form

$$H(e^{j\omega}) = e^{j\alpha(\omega)} F(\omega) \ ,$$

where $\alpha(\omega)$ and $F(\omega)$ are real-valued.

(ii) (4 pts.) If the input to the filter is the sequence

$$x[n] = 1 + 2^{-n} \qquad \text{(all } n) \ ,$$

determine the output $y[n]$.

(iii) (6 pts.) Now consider a *different* input sequence $x[n]$, which is periodic with (fundamental) period $L_x = 4$ samples. Determine the (fundamental) period L_y of the output sequence $y[n]$, *explaining your answer.*

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

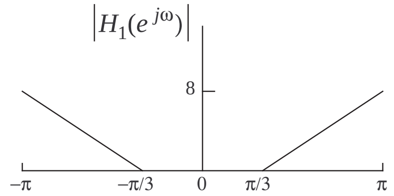
....

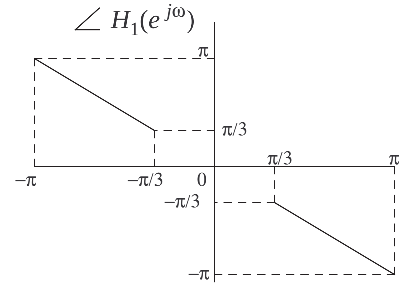
Leave blank →

LAST NAME: **First Name:**

PROBLEM 4 (15 pts.) **Grade:**

Consider a filter whose frequency response for $\omega \in (-\pi, \pi]$ is as shown below.





The filter accepts the input sequence

$$x[n] = \cos\left(\frac{\pi n}{7} - \frac{3\pi}{5}\right) + 3\cos\left(\frac{\pi n}{2}\right)$$

and produces the output sequence $v[n]$.

(i) (7 pts.) Determine $v[n]$.

(ii) (4 pts.) Are $x[n]$ and $v[n]$ periodic, and if so, what is the fundamental period in each case? *Explain.*

(iii) (4 pts.) The sequence $v[n]$ is the input to a second filter with impulse response

$$h_2[n] = \delta[n] - \delta[n-1]$$

Determine the output $y[n]$ of that (second) filter, *simplifying your answer as much as possible.*

Leave blank →

LAST NAME: **First Name:**

PROBLEM 5 (15 pts.) **Grade:**

Consider the following MATLAB code:

```
h = [ 3  -4   0   4  -3 ].' ;
s = [ 1   2   3   2   1 ].'    ;
H = fft(h,9)      ;
S = fft(s,9)      ;
C = H.*S          ;
c = ifft(C)       ;
```

(i) (8 pts.) Determine the vector `c`.

(ii) (7 pts.) *Without performing a convolution*, determine the vector `y` obtained by the following code:

```
b = [-3   4   0  -4   3 ].' ;
x = [ 1   2   3   3   3   3   2   1].' ;
B = fft(b,12) ;
X = fft(x,12) ;
Y = B.*X      ;
y = ifft(Y)   ;
```

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .