## ENEE 222: FINAL EXAM REVIEW

2. All signals shown below are periodic and have complex Fourier series expansions of the form

$$
s(t)=\sum_{k=-\infty}^{\infty} S_{k} e^{j k \Omega_{0} t}
$$

where $\Omega_{0}$ is the fundamental angular frequency.

(i) Assuming that $t$ is in seconds, what is the value of $\Omega_{0}$ ?
(ii) Evaluate $R_{0}$.
(iii) Express each $X_{k}$ in terms of $R_{k}$ 's.
(iv) Express each $Y_{k}$ in terms of $R_{k}$ 's. Simplify your answer as much as possible.
3. All signals shown below are periodic with complex Fourier series as in Problem 2 above. The curved segments of $y(t)$ are sinusoidal.

(i) Using the fact that $R_{k}=\sin (k \alpha \pi) /(k \pi)$, obtain an expression for each $X_{k}$. You need not simplify your answer.
(ii) Express each $Y_{k}$ in terms of $X_{k}$ 's.
4. A FIR filter has impulse response

$$
h[n]=b_{0} \delta[n]+b_{1} \delta[n-1]+b_{2} \delta[n-2]+b_{3} \delta[n-3]+b_{4} \delta[n-4]
$$

Its magnitude (amplitude) response is given by

$$
\left|H\left(e^{j \omega}\right)\right|=|\cos 2 \omega-2 \cos \omega|
$$

and its phase response is piecewise linear.
(i) Assuming that $b_{0}>0$, determine the values of $b_{0}, \ldots, b_{4}$.
(ii) Determine the response of the filter to the input

$$
x[n]=\left(\frac{2}{3}\right)^{n}, \quad n \in \mathbf{Z}
$$

(iii) The phase response $\angle H\left(e^{j \omega}\right)$ of the filter has exactly one discontinuity (jump) in the frequency interval $[0, \pi]$. At what value of $\omega$ does the discontinuity occur? Solve for $\omega$ algebraically (not graphically), using the identity $\cos 2 \omega=2 \cos ^{2} \omega-1$.
5. Consider the signal sequence $\mathbf{x}$ defined by

$$
x[n]=3 \cos \left(\frac{\pi n}{3}+\frac{2 \pi}{5}\right)+2 \cos (2.75 n-1.24), \quad n \in \mathbf{Z}
$$

(i) If $\mathbf{x}$ is the input to a filter with magnitude (amplitude) and phase responses as shown in figure below, determine the resulting output sequence $\mathbf{y}$.
(ii) Is the output $\mathbf{y}$ periodic, and if so, what is its period?


7. Two FIR filters whose impulse response sequences $\mathbf{h}^{(1)}$ and $\mathbf{h}^{(2)}$ are given by

$$
h^{(1)}[n]=\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]+\delta[n-4]
$$

and

$$
h^{(2)}[n]=\delta[n]-\delta[n-1]
$$

are connected in series (cascade) to form a single filter with impulse response $\mathbf{h}$.
(i) Using $z$-transforms or convolution, determine $h[n]$ for every $n$.
(ii) I claim that if the input to the cascade is a periodic sequence $\mathbf{x}$ of period $N=5$, then the output sequence $\mathbf{y}$ is constant in value, i.e., $y[n]=c$ for all $n \in \mathbf{Z}$; and that, furthermore, the value $c$ is the same for all periodic sequences of period $N=5$.
Is my claim correct? If it is correct, what is the value of $c$ ? Explain.

