04/12 REVIEW

PROBLEM 8

The continuous-time signal

$$x(t) = -4 + 5\cos(700\pi t + 0.2) + 2\cos(450\pi t - 0.8)$$

is sampled every $T_s = 0.001$ seconds starting at $t = 0$. The first $N = 40$ samples are stored in the vector $\mathbf{x}$.

(i) Verify that the frequencies of the sinusoidal components of $x(t)$ become (after sampling) Fourier frequencies for the N-point vector $\mathbf{x}$.

(ii) Compute the nonzero entries in the DFT $\mathbf{X}$ of $\mathbf{x}$. Give your answers in either Cartesian or polar form, clearly showing the corresponding indices k.

(iii) Sketch the amplitude and phase spectra $|X[k]|$ and $\angle X[k]$ as functions of the frequency index $k = 0, \ldots, 39$.

PROBLEM 2

The time-domain signal

$$\mathbf{s} = \begin{bmatrix} a & b & c & d & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

has DFT given by

$$\mathbf{S} = \begin{bmatrix} 4 & 4 - j(2-\sqrt{2}) & 2 & 4 + j(2+\sqrt{2}) & 8 & 4 - j(2+\sqrt{2}) & 2 & 4 + j(2-\sqrt{2}) \end{bmatrix}^T$$

(i) Compute the numerical value of $a - b + c - d$.

(ii) Compute the DFT $\mathbf{X}$ of the time-domain signal

$$\mathbf{x} = \begin{bmatrix} a & b & c & d & a & b & c & d \end{bmatrix}^T$$

(iii) Compute the DFT $\mathbf{Y}$ of the time-domain signal

$$\mathbf{y} = \begin{bmatrix} a & b & c & d & -a & -b & -c & -d \end{bmatrix}^T$$

PROBLEM 4

The time-domain signals

$$\mathbf{x} = \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix}^T \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} 3 & -1 & 4 & -2 \end{bmatrix}^T$$

have DFT's given by

$$\mathbf{X} = \begin{bmatrix} A & B & C & D \end{bmatrix}^T \quad \text{and} \quad \mathbf{Y} = \begin{bmatrix} E & F & G & H \end{bmatrix}^T$$

(i) Compute the time-domain signal $\mathbf{s}$ whose DFT is given by

$$\mathbf{S} = \begin{bmatrix} AE & BH & CG & DF \end{bmatrix}^T$$

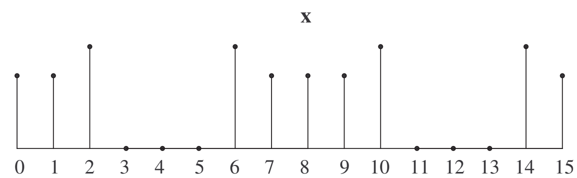
(ii) Let $\mathbf{u}$ be the time-domain signal defined by the circular convolution

$$\mathbf{u} = \begin{bmatrix} A & B & C & D \end{bmatrix}^T \circledast \begin{bmatrix} 3 & -1 & 4 & -2 \end{bmatrix}^T$$

Write an expression for the DFT $\mathbf{U}$ of $\mathbf{u}$ in terms of E, F, G and H.

PROBLEM 5

The time-domain signal $\mathbf{x}$ shown below takes three distinct values (zero being one of them) and has DFT $\mathbf{X}$. For simplicity, let $X[k] = X_k$.

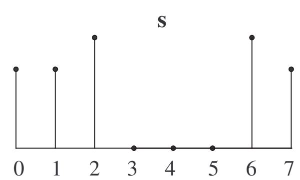


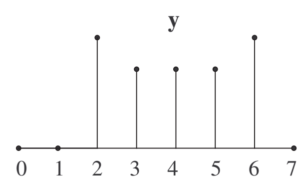
(i) For which values of k (if any) does X_k equal 0?

(ii) For which pairs (k, k') (if any) does X_k equal $X_{k'}$?

(iii) Express the DFT $\mathbf{S}$ of the time-domain signal $\mathbf{s}$ shown below (left) in terms of X_k's.

(iv) Express the DFT $\mathbf{Y}$ of the time-domain signal $\mathbf{y}$ shown below (right) in terms of X_k's.





PROBLEM 6

The DFT of the signal

$$\mathbf{x} = \begin{bmatrix} a & b & c & d & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

is given by

$$\mathbf{X} = \begin{bmatrix} X_0 & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 \end{bmatrix}^T$$

(i) Determine the DFT $\mathbf{S}$ of the time-domain signal

$$\mathbf{s} = \begin{bmatrix} c & d & a & b \end{bmatrix}^T$$

(ii) Compute the value of $\cos(n\pi/2)$ for $n = 0, \ldots, 7$.

(iii) Using your answer to part (ii) above, determine the time-domain signal $\mathbf{y}$ whose DFT is given by

$$\mathbf{Y} = \begin{bmatrix} X_0 & 0 & -X_2 & 0 & X_4 & 0 & -X_6 & 0 \end{bmatrix}^T$$