EXAM 1 REVIEW

PROBLEM 1

Let $z = x + jy$ and $w = e^{j\theta}$, where x, y and θ are real-valued.

(i) Express $z + z^{-1}$ in Cartesian form.

(ii) Express $w^3 + w^{-3}$ as a real-valued function of θ.

(iii) Express $|z^* - w|^2$ as a sum of real-valued terms involving x,y and θ.

(iv) Express z/w in Cartesian form.

PROBLEM 6

(i) On the complex (z) plane, sketch the lines given by the following equations:

- $|z - 1| \;=\; |z - 2j|$
- $|z - 1 - j| \;=\; \sqrt{2}$

(ii) The sinusoid

$$x(t) = A\cos(\Omega t + \pi/4) + B\sin(\Omega t + \pi/3)$$

can be expressed as

$$x(t) = C\cos(\Omega t + \phi)$$

for suitable C and ϕ. Find an equation for C^2 in terms of A and B. (You do not need to solve for ϕ).

$$(\ \cos(\pi/6) = \sin(\pi/3) = \sqrt{3}/2; \quad \cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2; \quad \cos(\pi/3) = \sin(\pi/6) = 1/2\ )$$

PROBLEM 2

A continuous-time signal $x(t)$ is a sum of two sinusoids whose frequencies are known to lie in the range 500 Hz – 550 Hz. It is sampled at a rate of 100 samples/second starting at $t = 0$. The resulting samples satisfy the equation

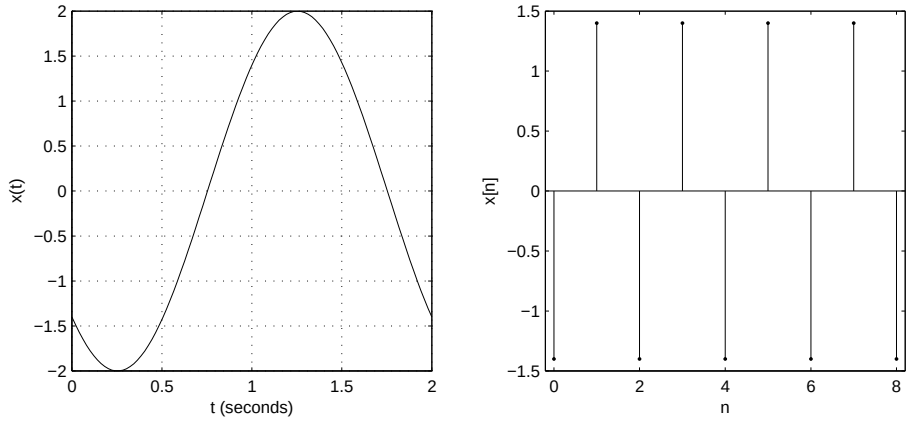
$$x[n] = 3.4\cos(0.3\pi n + 1.7) + 2.1\cos(0.8\pi n + 2.5)$$

(i) Find an equation for $x(t)$.

(ii) How would your answer to **(i)** differ if the frequencies of the two components of $x(t)$ were in the range 550 Hz – 600 Hz instead?

PROBLEM 5

The real-valued sinusoid $x(t)$ has period 2.0 seconds and amplitude 2.0 units. It is plotted below (left) for t in $[0.0, 2.0]$.



(i) Calculate the total amount of time, over one period, for which $x(t) \geq 1.0$.

(ii) If $x(0) = -\sqrt{2}$, determine the exact time t in the above graph such that $x(t) = 2.0$.

(iii) The sinusoid $x(t)$ is sampled every T_s seconds to produce the discrete-time signal

$$x[n] = x(nT_s)$$

shown above, on the right, for $n = 0, \ldots, 8$. Determine *all* possible values of T_s.

PROBLEM 7

A real-valued sinusoid $x(t)$ has the following features:

- It is rising (i.e., its value is increasing) at time $t = 0$.
- Its first peak after $t = 0$ is observed at $t = 0.1$ sec.
- The value of that maximum equals 2.0 volts.
- The first zero value of $x(t)$ is observed at $t = 0.4$ sec.

(i) What is the period of $x(t)$?

(ii) What is the value of $x(0)$?

(iii) Write an equation for $x(t)$.

(iv) Suppose we sample the sinusoid every $T_s = 0.1$ seconds, starting at time $t = 0$. Will the value $x(0)$ occur again (as a sample) *after* time 0? If so, when will it first reoccur?