# EXAM 1 REVIEW (03/03/20)

## PROBLEM 1

**(i)** On the complex plane, sketch the curve given by the equation

$$|z - j| = 2$$

**(ii)** Solve the equation

$$z^3 = j$$

Express the roots in Cartesian form and plot them on the complex plane. Which of these roots lie on the curve found in part **(i)**?

*Part (iii) is unrelated to parts (i) and (ii).*

**(iii)** A discrete-time linear filter accepts the input

$$x[n] = \cos\left(\frac{2\pi n}{3}\right)$$

and produces the output

$$y[n] = x[n-1] + x[n] + x[n+1]$$

Show that $y[n] = 0$ at all times $n$.

## PROBLEM 2

Consider the real-valued sinusoid $x(t) = A\cos(\Omega t + \phi)$, where $A > 0$ and $\phi \in [0, 2\pi)$.

It is known that $x(0) = 1$, and that the first two zeros (i.e., zero-crossings) in positive time occur at $t = 0.12$ and $t = 0.30$ sec.

**(i)** Sketch the signal over a time interval that includes at least one period, adding features and values obtained below.

**(ii)** Determine the period $T$, as well as the angular frequency $\Omega$ (rad/sec).

**(iii)** Determine the initial phase $\phi$.

**(iv)** Determine the amplitude $A$. If need be, express your answer in terms of $\cos(\theta)$ (for some $\theta$).

**PROBLEM 3**

Consider the discrete-time sinusoid

$$v[n] \;=\; 2\cos\left(\frac{7\pi n}{12} - 1.3\right)$$

**(i)** What is the smallest *integer* $N$ such that $v[n+N] = v[n]$ for all $n$? If no such integer exists, explain why.

**(ii)** Suppose that $v[n]$ is obtained by sampling a continuous-time sinusoid $x(t) \;=\; 2\cos(2\pi f_1 t + \phi_1)$ at rate $f_s = 360$ samples/sec. If $f_1$ is known to lie in the frequency range [720, 900] Hz, write an equation for $x(t)$.

**(iii)** Can you find two *distinct* frequencies $f_2$ and $f_3$ in the frequency range [0, 360] Hz such that the continuous-time signal

$$y(t) \;=\; \cos(2\pi f_2 t + 1.3) \;+\; \cos(2\pi f_3 t - 1.3)$$

also produces $v[n]$ (as defined above) when sampled at the same rate $f_s = 360$ samples/sec? If so, determine those frequencies. Otherwise, explain why they do not exist.

**PROBLEM 4**

Consider the continuous-time sinusoid

$$x(t) \;=\; \cos(20\pi t \;-\; 1.7)\ ,$$

where $t$ is in seconds.

**(i)** Determine the period $T$ of $x(t)$.

**(ii)** Determine all values of $T_s$ greater than 0 and less than $T$ such that

$$x[n] \;=\; x(nT_s)$$

is periodic with fundamental period $N = 10$ samples.

**(iii)** Consider also the sinusoid

$$y(t) \;=\; \cos(70\pi t \;+\; 1.7)$$

What are the *two highest* positive sampling rates $f_a = 1/T_a$ such that

$$y(nT_a) \;=\; x(nT_a)$$

for all indices $n$?