Lecture 24

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- Block convolution; in-class assignment
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- Practical FIR filters

Linear Convolution via Circular Convolution

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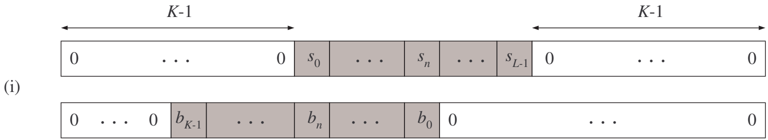

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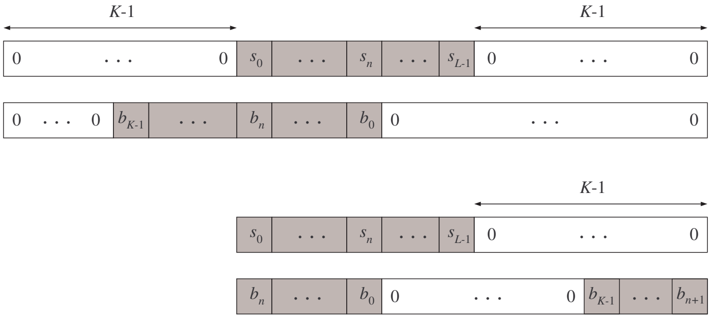
(i)

(ii)



Convolution and Polynomial Multiplication

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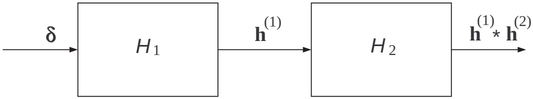

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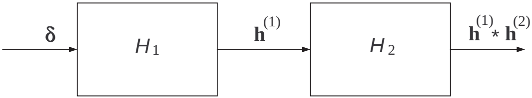
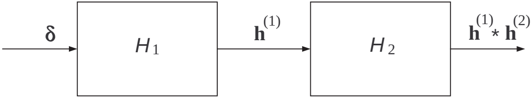


- But also:

$$H(z) \;=\; H_1(z)H_2(z)$$

Convolution and Polynomial Multiplication



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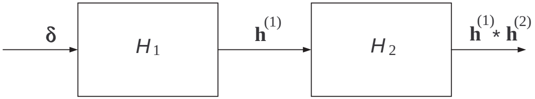
- Example from last class:

$$\begin{bmatrix} 2 & -1 & 1 & -2 \end{bmatrix}^T * \begin{bmatrix} 1 & 2 & 3 & -1 \end{bmatrix}^T$$

Convolution and Polynomial Multiplication



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- Example from last class:

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can be also computed via

$$(2 - z^{-1} + z^{-2} - 2z^{-3})(1 + 2z^{-1} + 3z^{-2} - z^{-3})$$

In-Class Assignment

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An FIR filter acts on the input sequence given by

$$x[0:5] \; = \; \left[\begin{array}{cccccc} 1 & -3 & 5 & -2 & 6 & 2 \end{array} \right]^T$$

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Without performing a convolution, determine the response $v[\cdot]$ of the filter to the input sequence $u[\cdot]$ which equals zero except for

$$u[0:9] \;=\; \left[\begin{array}{cccccccccc} 2 & -6 & 10 & -4 & 11 & 7 & -5 & 2 & -6 & -2 \end{array}\right]^T$$

Ideal Filter in Frequency Domain

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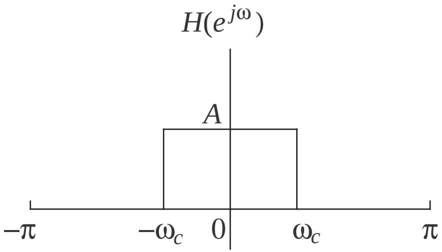
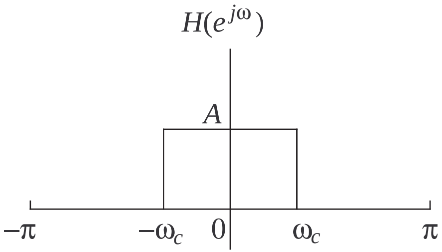


Ideal Filter in Frequency Domain



Cannot be realized in practice

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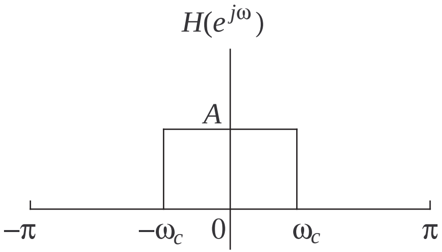

Cannot be realized in practice (it is inherently noncausal)

Practical Filter Characteristics

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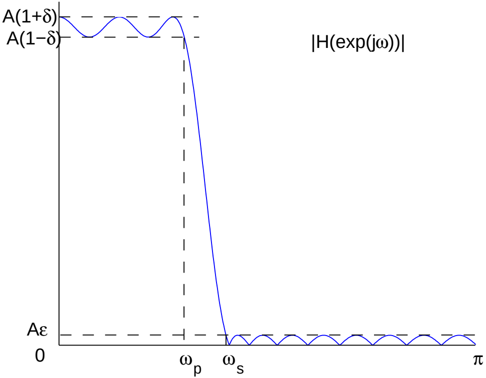

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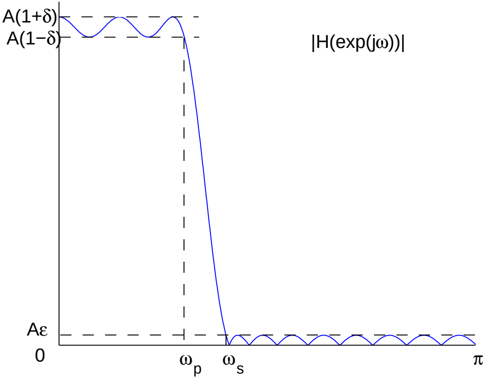
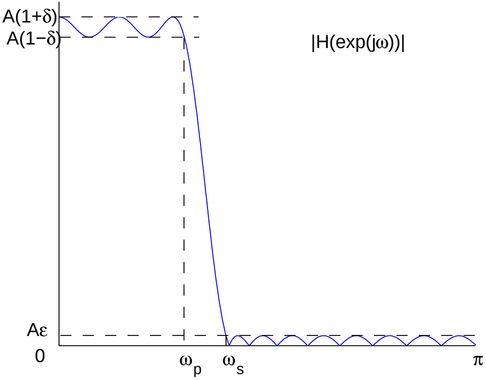


- Transition band $[\omega_p, \omega_s]$

Practical Filter Characteristics



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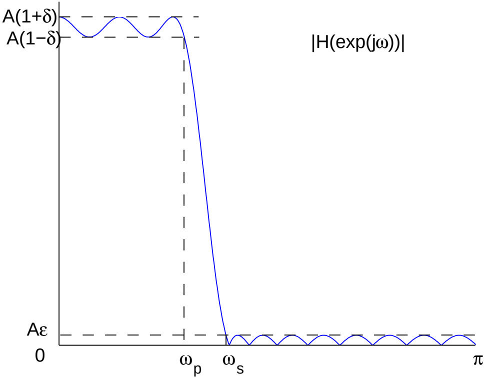
Practical Filter Characteristics



- Transition band $[\omega_p, \omega_s]$
- Passband ripple δ
- Stopband attenuation $1/\epsilon$