Overview

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  - complex exponentials

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Finite-Duration Input

Consider the general FIR filter

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Consider the general FIR filter of order M

Finite-Duration Input

Consider the general FIR filter of order M, with coefficient vector

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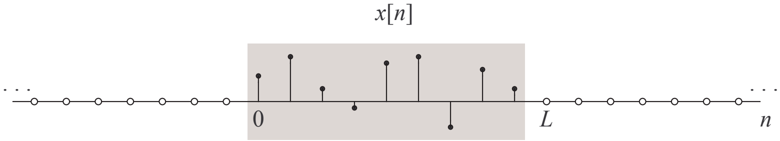

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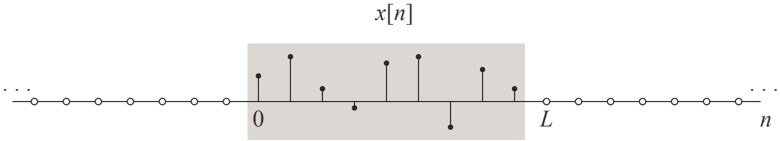

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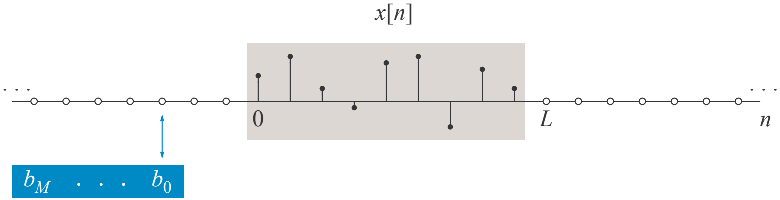

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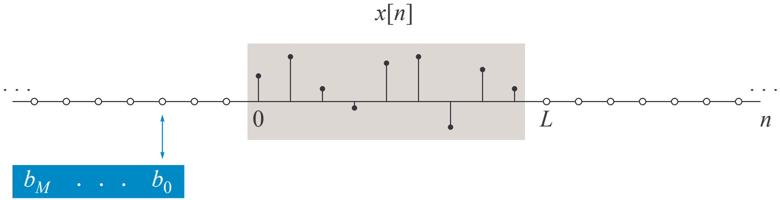

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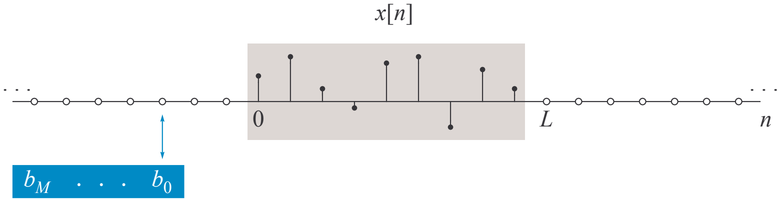

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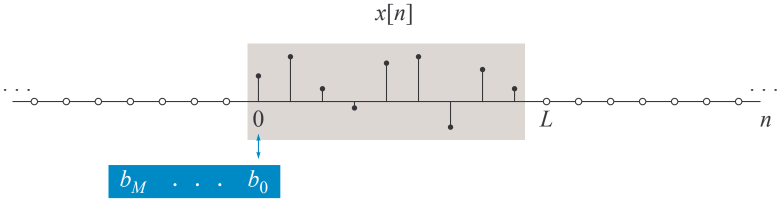

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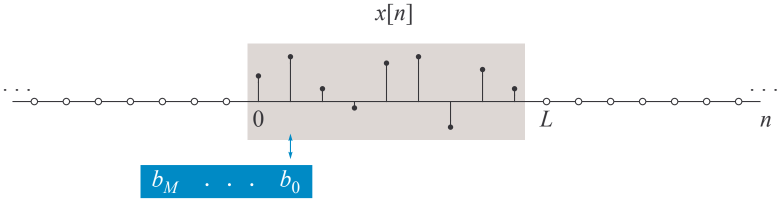

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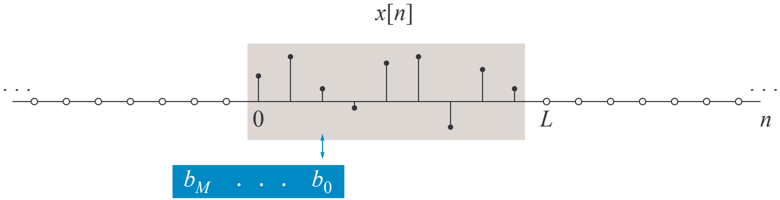

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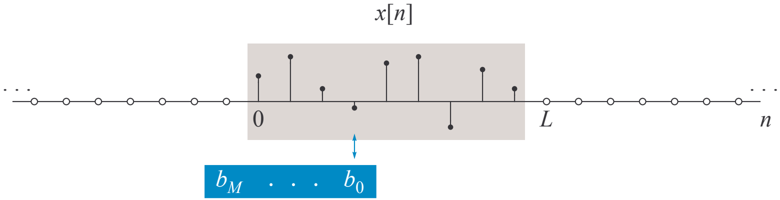

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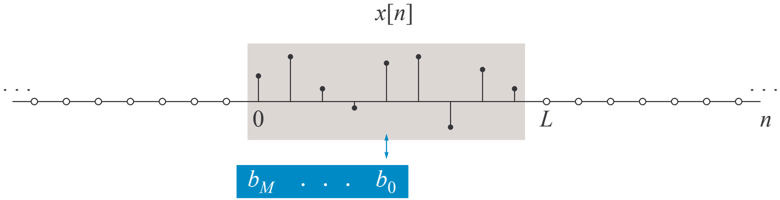

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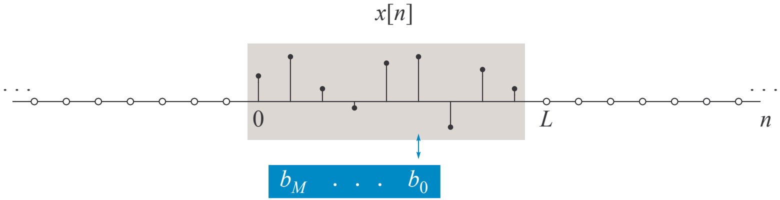

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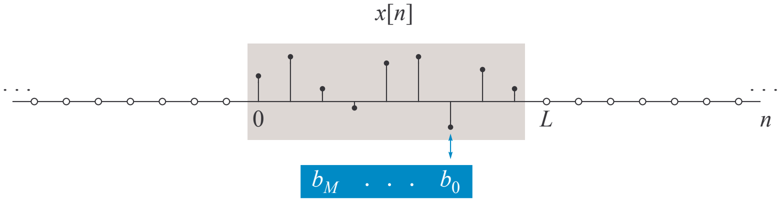

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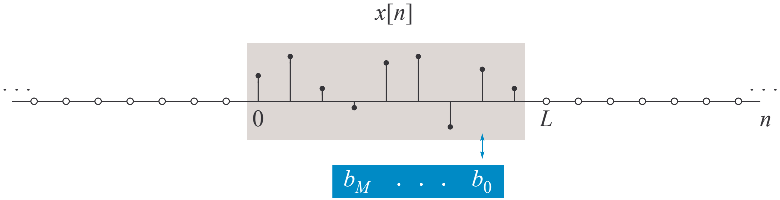

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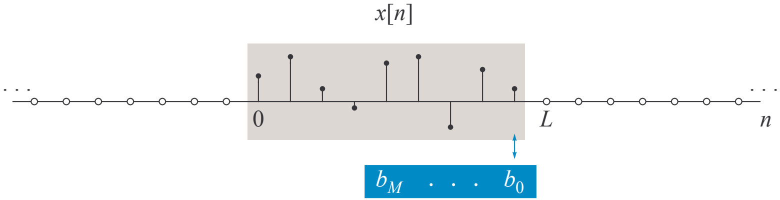

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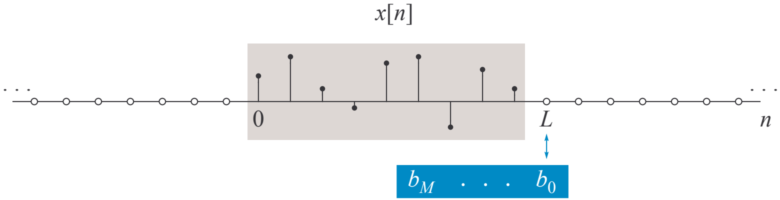

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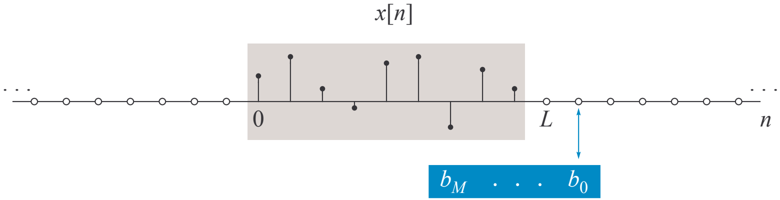

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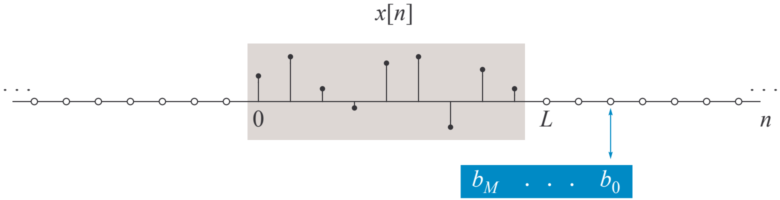

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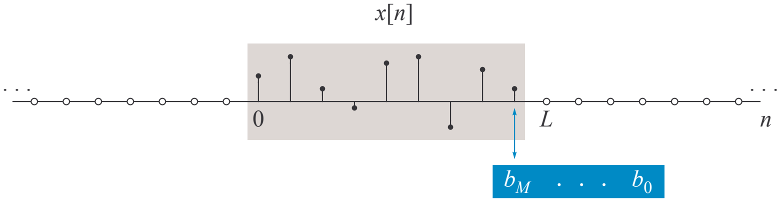

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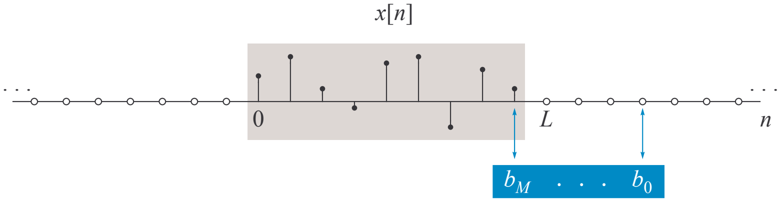

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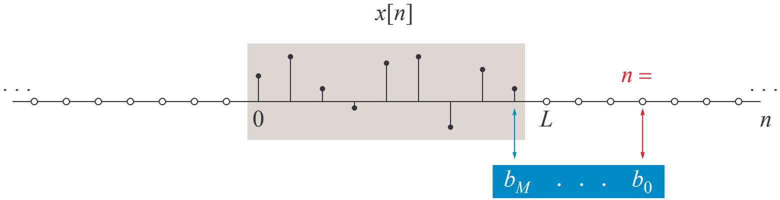

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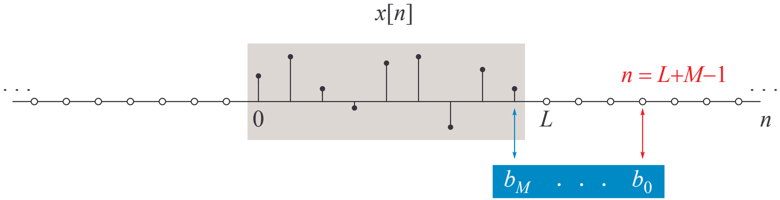

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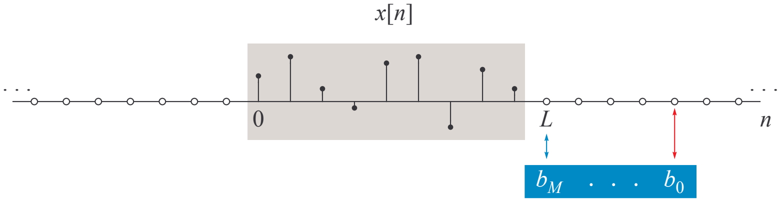

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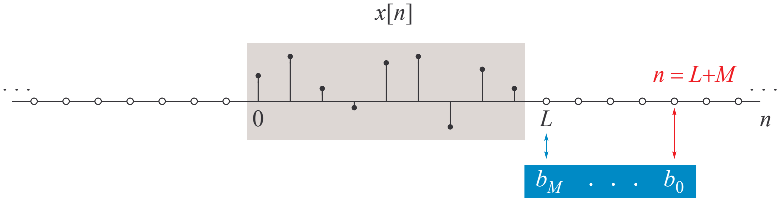

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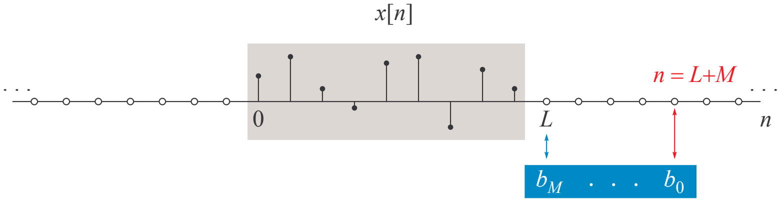

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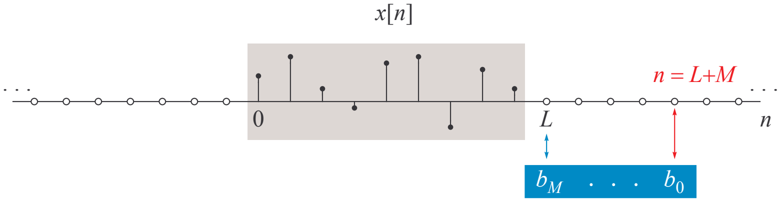

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Nontrivial portion of output:

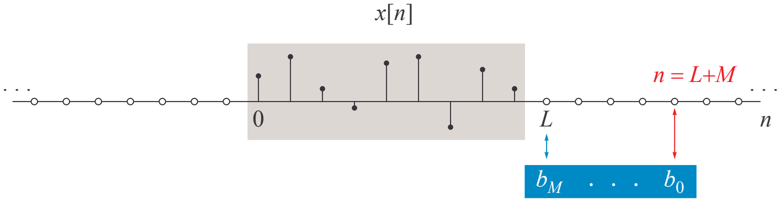
Finite-Duration Input

Consider the general FIR filter of order M, with coefficient vector

$$\mathbf{b} = [\, b_0 \;\; b_1 \;\; \ldots \;\; b_M]^T \,,$$

where $b_0 b_M \neq 0$.

Input sequence $x[\,\cdot\,]$ has finite duration. Its nontrivial samples are $x[0 : L-1]$, where $x[0]\cdot x[L-1] \neq 0$.



$$y[n] = 0 \quad \text{for} \quad n < 0 \quad \text{and} \quad n \geq L + M$$

Nontrivial portion of output: $y[0 : L + M - 1]$

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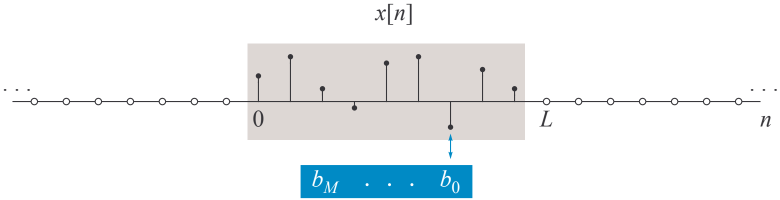

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Properties of Linear Convolution

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$$\mathbf{b} * \mathbf{s}$$

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Implementation via Circular Convolution

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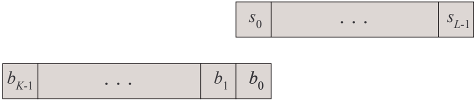

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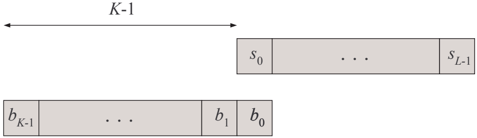

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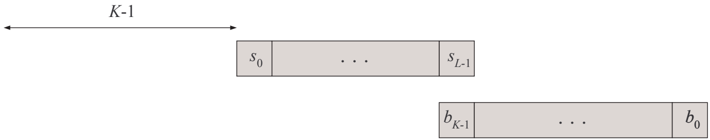

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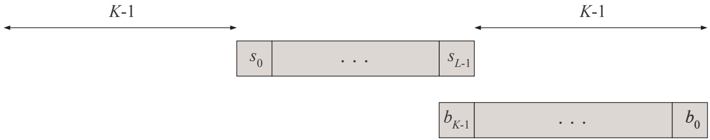

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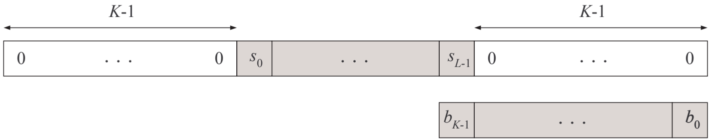

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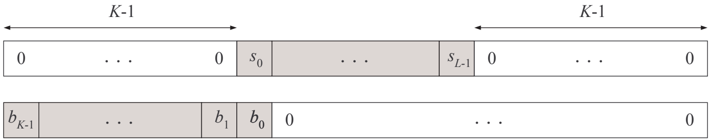

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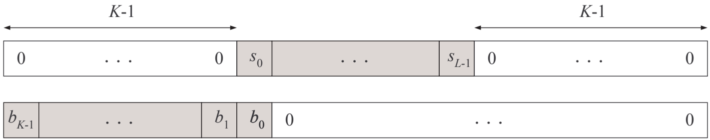

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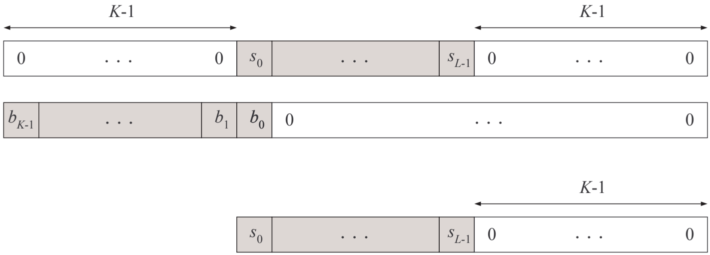

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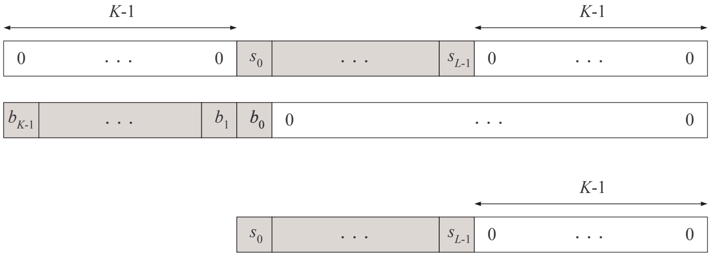

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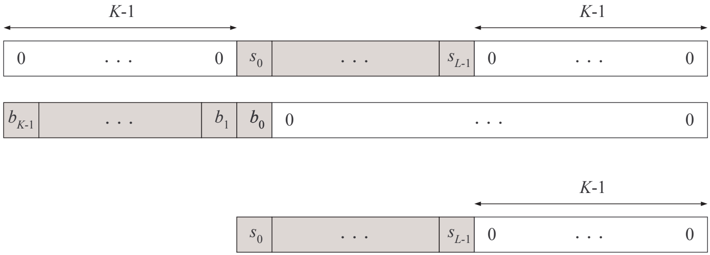

Recall:

Implementation via Circular Convolution

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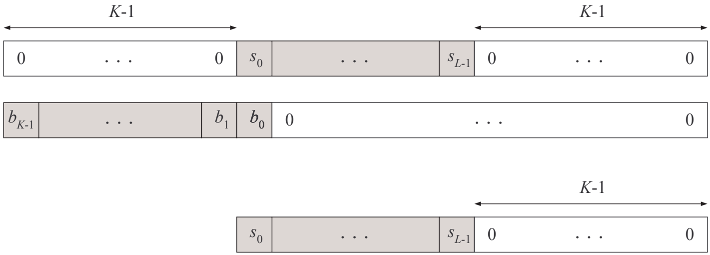

Recall: i^{th} entry of $\mathbf{x} \circledast \mathbf{y}$

Implementation via Circular Convolution

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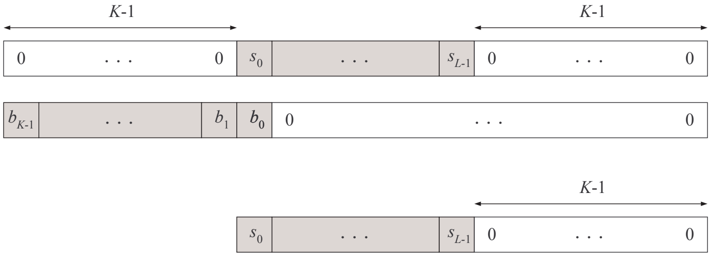

Recall: i^{th} entry of $\mathbf{x} \circledast \mathbf{y}$ = dot product

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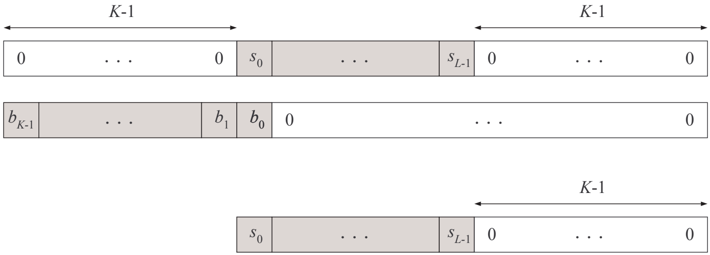

Recall: i^{th} entry of $\mathbf{x} \circledast \mathbf{y} \;=\;$ dot product of $\mathbf{P}^i(\mathbf{R}\mathbf{x})$

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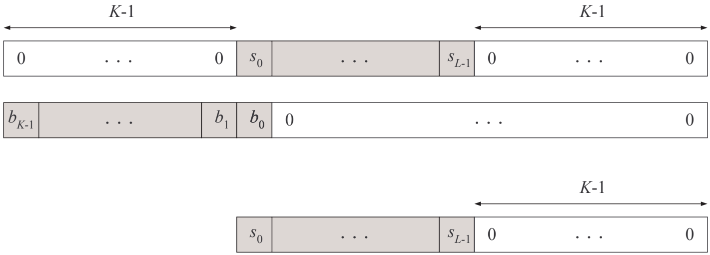

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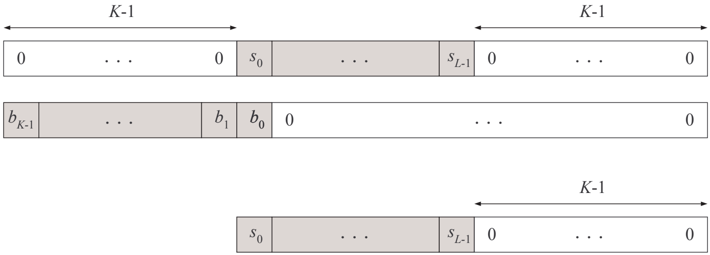

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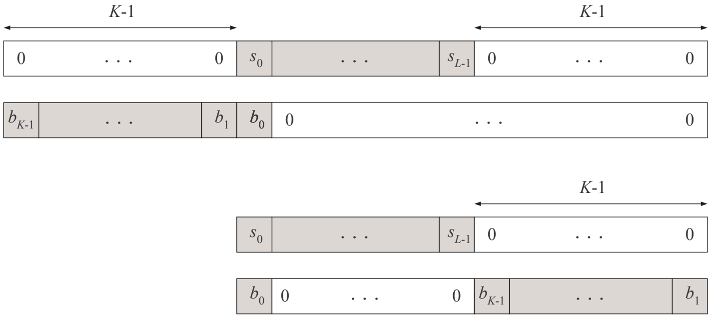

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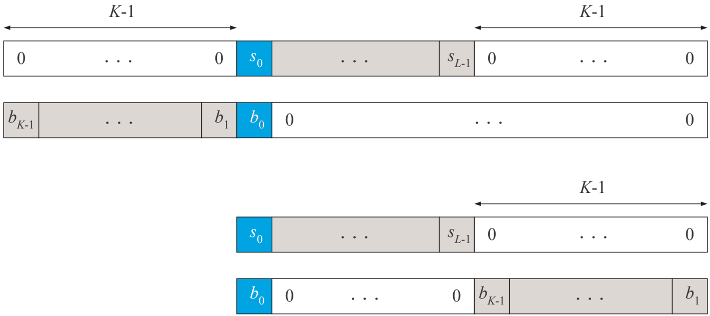

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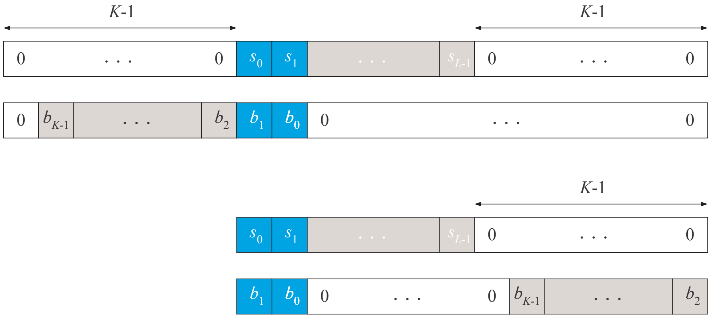

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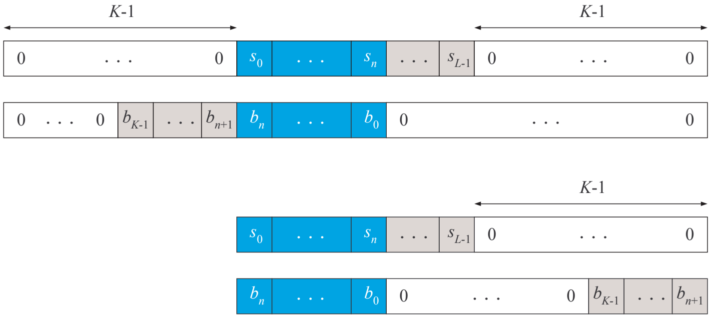

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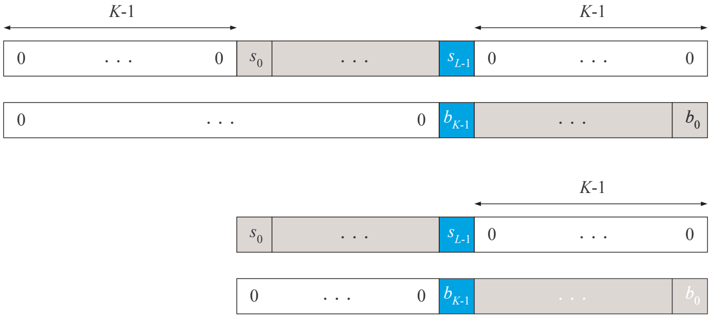

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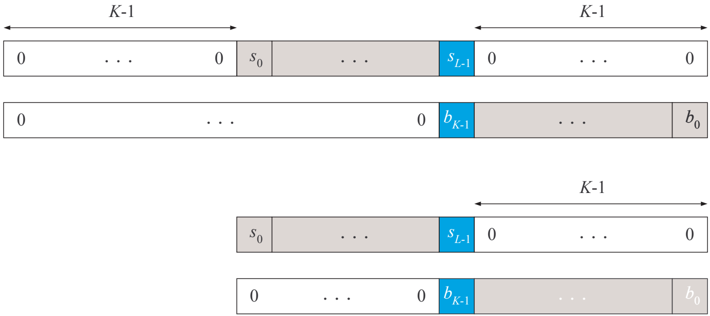

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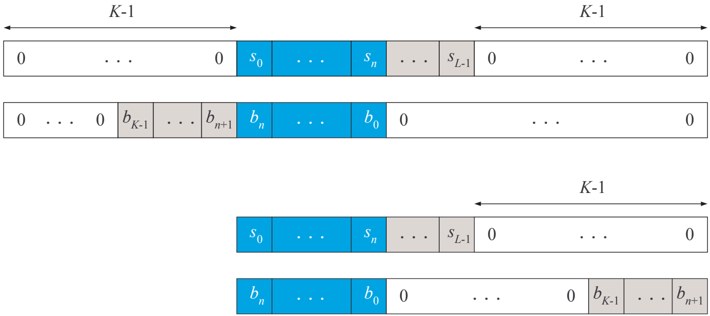

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