Lecture 23

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- FIR filters with arbitrary inputs

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- FIR filters with arbitrary inputs; impulse response

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- FIR filters with arbitrary inputs; impulse response, linear convolution

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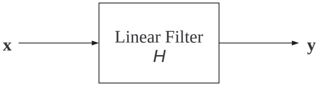
- Linear convolution of vectors

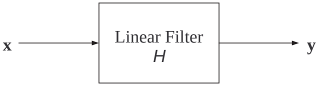
Lecture 23

- FIR filters with arbitrary inputs; impulse response, linear convolution
- Linear convolution of vectors
- Relationship between linear and circular convolution

FIR Filters

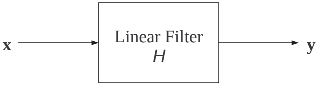


FIR Filters



- Input-output relationship:

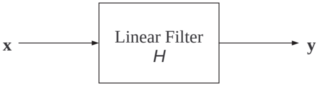
FIR Filters



- Input-output relationship:

$$(\forall n) \qquad y[n] \;=\; b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

FIR Filters

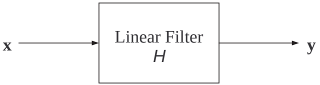


- Input-output relationship:

$$(\forall n) \qquad y[n] \;=\; b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

- System function: $H(z) = b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}$

FIR Filters



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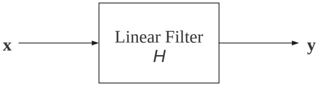
$$x[n] \;=\; e^{j\omega n}$$

FIR Filters



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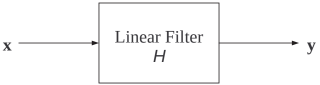
$$x[n] \;=\; e^{j\omega n} \;\;\Rightarrow\;\; y[n] \;=\; H(e^{j\omega}) x[n]$$

FIR Filters



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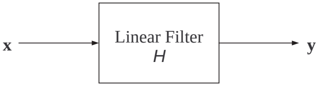
$$x[n] \;=\; e^{j\omega n} \quad \Rightarrow \quad y[n] \;=\; H(e^{j\omega})x[n]$$

$$x[n] \;=\; \cos(\omega n)$$

FIR Filters



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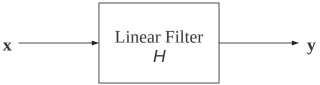
$$x[n] = e^{j\omega n} \quad \Rightarrow \quad y[n] = H(e^{j\omega}) x[n]$$

$$x[n] = \cos(\omega n) \quad \Rightarrow \quad y[n] = |H(e^{j\omega})| \cos(\omega n + \angle H(e^{j\omega}))$$

FIR Filters



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$$\begin{aligned} x[n] &= e^{j\omega n} &\Rightarrow\quad y[n] &= H(e^{j\omega})x[n] \\ x[n] &= \cos(\omega n) &\Rightarrow\quad y[n] &= |H(e^{j\omega})|\cos(\omega n + \angle H(e^{j\omega})) \\ x[n] &= z^n \end{aligned}$$

FIR Filters

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$$x[n] = e^{j\omega n} \Rightarrow y[n] = H(e^{j\omega}) x[n]$$

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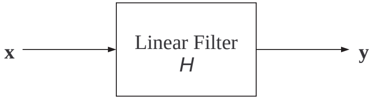
$$x[n] = z^n \Rightarrow y[n] = H(z) z^n$$

FIR Filters



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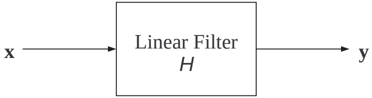

- If the input $x[\cdot]$ is *periodic* with period L, then so is the output $y[\cdot]$.

FIR Filters

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  Circular convolution can then be used to compute $y[0 : L - 1]$.

Properties of Linear Convolution (Vectors)

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$$\mathbf{b} * [\mathbf{0}_i\,;\,\mathbf{s}] \;=\; [\mathbf{0}_i\,;\,\mathbf{b} * \mathbf{s}]$$

Properties of Linear Convolution (Vectors)

$$\begin{aligned}\mathbf{b} * [\mathbf{0}_i\,;\,\mathbf{s}] &= [\mathbf{0}_i\,;\,\mathbf{b} * \mathbf{s}] \\ \mathbf{b} * [\mathbf{s}\,;\,\mathbf{0}_i] &= [\mathbf{b} * \mathbf{s}\,;\,\mathbf{0}_i]\end{aligned}$$

Properties of Linear Convolution (Vectors)

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\mathbf{b} * [\mathbf{0}_i\,;\, \mathbf{s}] &= [\mathbf{0}_i\,;\, \mathbf{b} * \mathbf{s}] \\
\mathbf{b} * [\mathbf{s}\,;\, \mathbf{0}_i] &= [\mathbf{b} * \mathbf{s}\,;\, \mathbf{0}_i] \\
\mathbf{b} * (\alpha \mathbf{s}) &= \alpha(\mathbf{b} * \mathbf{s})
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\mathbf{b} * (\mathbf{r} + \mathbf{s}) &= \mathbf{b} * \mathbf{r} + \mathbf{b} * \mathbf{s}
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If $\mathbf{b}$ and $\mathbf{s}$ have length K and L (respectively),

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If $\mathbf{b}$ and $\mathbf{s}$ have length K and L (respectively), then

$$
\mathbf{b} * \mathbf{s} = [\mathbf{b} \,;\, \mathbf{0}_{L-1}] \circledast [\mathbf{s} \,;\, \mathbf{0}_{K-1}]
$$

Linear Convolution via Circular Convolution

$$\mathbf{b} * \mathbf{s} \ = \ [\mathbf{b}\,;\,\mathbf{0}_{L-1}] \circledast [\mathbf{s}\,;\,\mathbf{0}_{K-1}]$$

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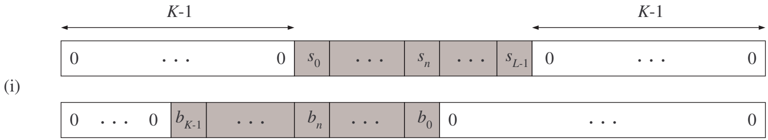

Linear Convolution via Circular Convolution

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