Bandlimited Case: Sampling

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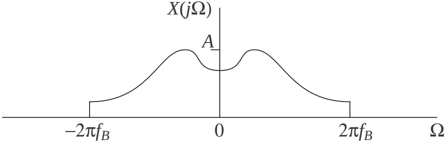
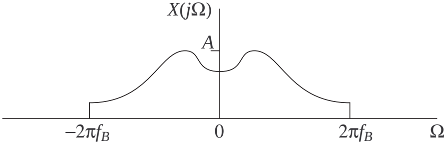
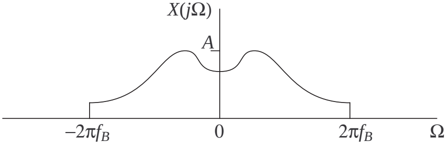


Bandlimited Case: Sampling



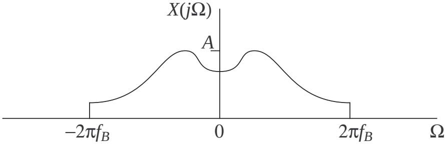
$$x[n] = x(n\Delta)$$

Bandlimited Case: Sampling



$$x[n] \;=\; x(n\Delta) \;=\; x(n/f_s)$$

Bandlimited Case: Sampling



$$x[n] = x(n\Delta) = x(n/f_s)$$

- $r = 2f_B/f_s < 1$:

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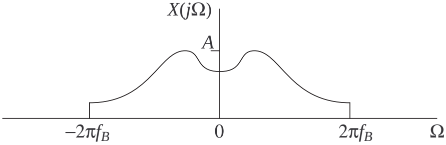

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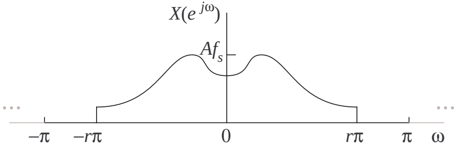

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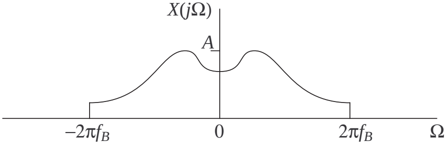

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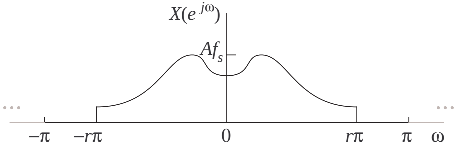
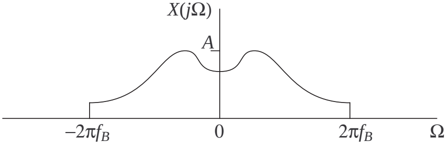


$$x(t) \;=\; \sum_{n=-\infty}^{\infty} x[n] \cdot \mathrm{sinc}((t/\Delta) - n)$$

Bandlimited Case: Sampling



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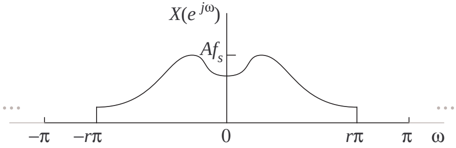

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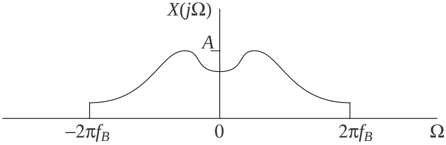
Bandlimited Case: Sampling



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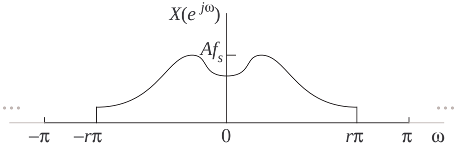
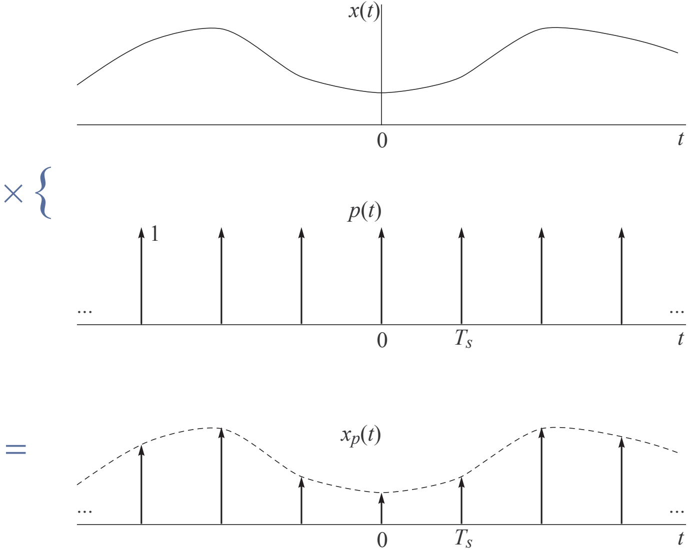


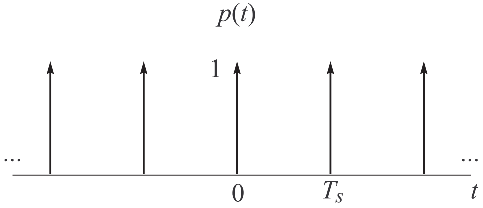
$$x(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot \mathrm{sinc}((t/\Delta) - n)$$

- $r = 2f_B/f_s > 1$: Interpolation impossible due to aliasing

Impulse Sampling in Time Domain



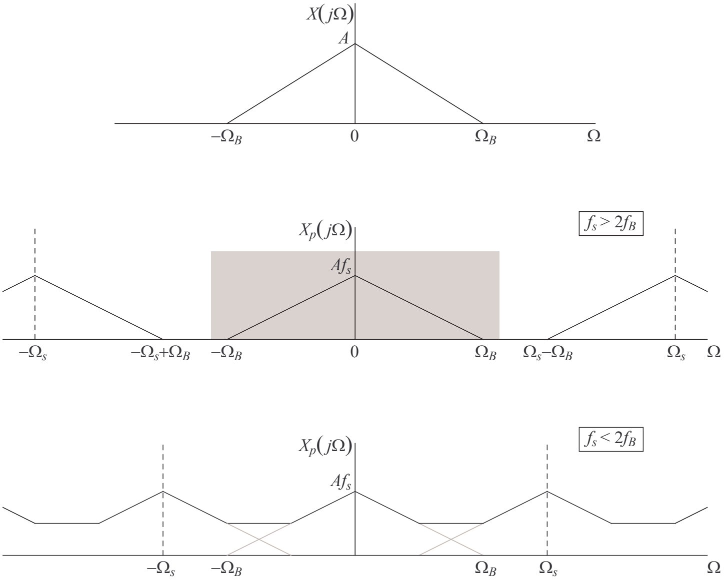
Periodic Impulse Train



$$p(t) \;=\; \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \;=\; f_s \cdot \sum_{k=-\infty}^{\infty} e^{jk\Omega_s t}$$

Fourier series: heuristic (does not converge)

Impulse Sampling in Frequency Domain



Passive Circuit: State-Space Model

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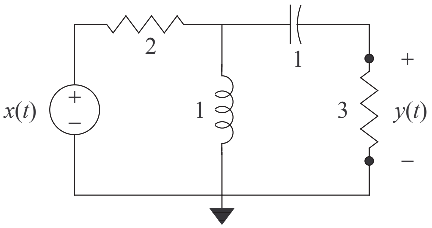
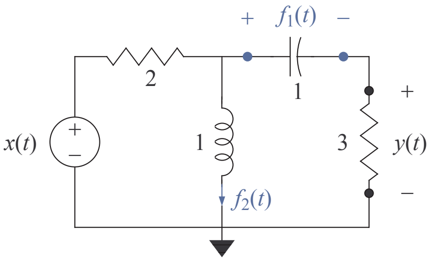


Passive Circuit: State-Space Model



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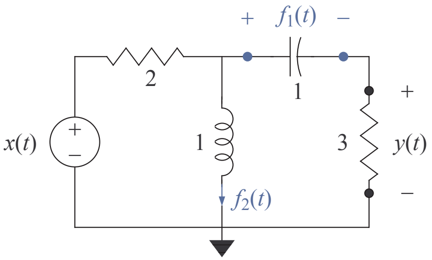
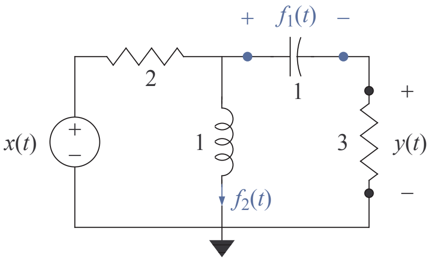


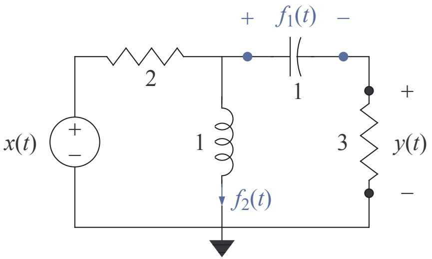
State variables:

Passive Circuit: State-Space Model



State variables: $f_1(t)$ and $f_2(t)$

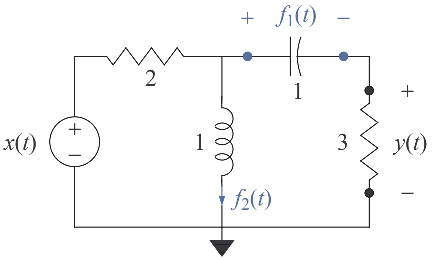
Passive Circuit: State-Space Model



State variables: $f_1(t)$ and $f_2(t)$

State equation:

Passive Circuit: State-Space Model

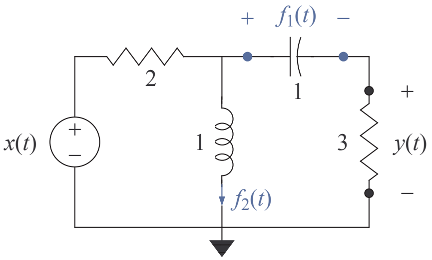


State variables: $f_1(t)$ and $f_2(t)$

State equation:

$$\begin{bmatrix} \dot{f}_1(t) \\ \dot{f}_2(t) \end{bmatrix} =$$

Passive Circuit: State-Space Model

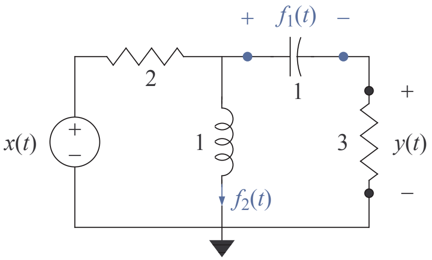


State variables: $f_1(t)$ and $f_2(t)$

State equation:

$$\begin{bmatrix} \dot{f}_1(t) \\ \dot{f}_2(t) \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -1 & -2 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

Passive Circuit: State-Space Model



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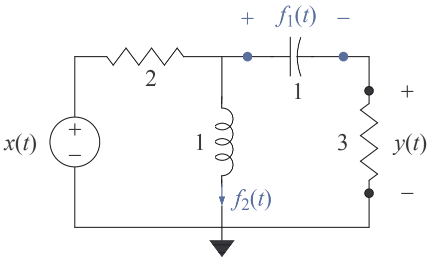
Passive Circuit: State-Space Model



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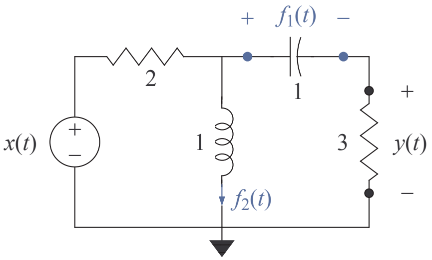
Output equation:

Passive Circuit: State-Space Model



State variables: $f_1(t)$ and $f_2(t)$

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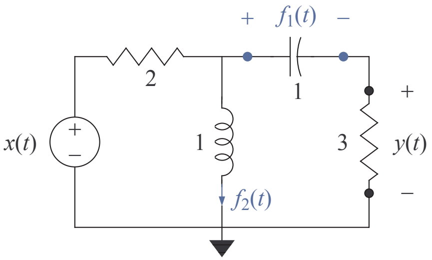
Output equation:

$$y(t) =$$

Passive Circuit: State-Space Model



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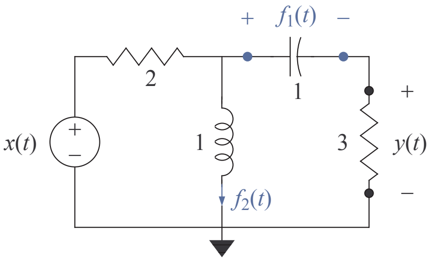
Output equation:

$$y(t) = \frac{3}{5}\begin{bmatrix} -1 & -2 \end{bmatrix}\begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

Passive Circuit: State-Space Model



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Output equation:

$$y(t) = \frac{3}{5}\begin{bmatrix} -1 & -2 \end{bmatrix}\begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} + \frac{3}{5}x(t)$$

Passive Circuit: Frequency Response

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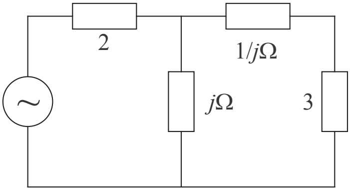

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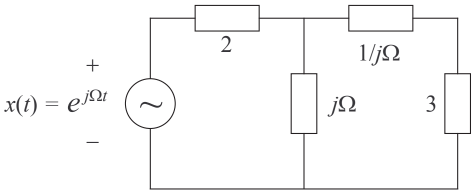

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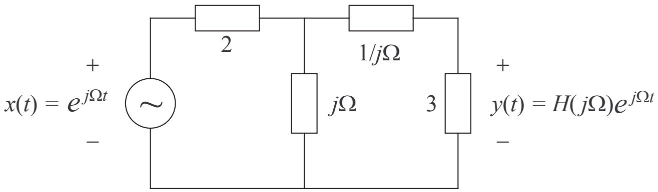
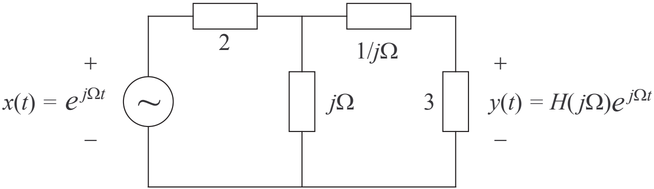
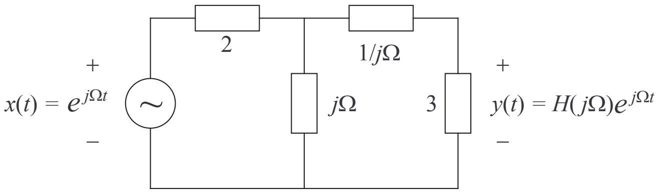


Passive Circuit: Frequency Response



- By AC steady-state analysis:

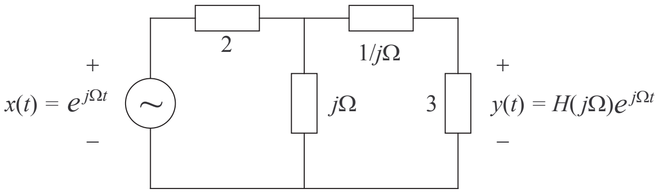
Passive Circuit: Frequency Response



- By AC steady-state analysis:

$$H(j\Omega) = \frac{3(j\Omega)^2}{5(j\Omega)^2 + 7(j\Omega) + 2}$$

Passive Circuit: Frequency Response



- By AC steady-state analysis:

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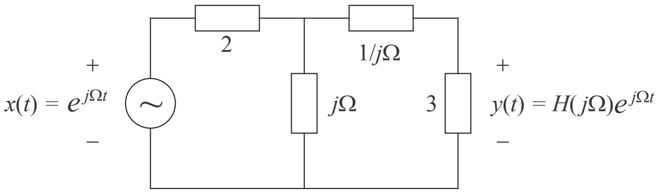
  - $$\left(5(j\Omega)^2 + 7(j\Omega) + 2\right) \cdot Y(j\Omega) = 3(j\Omega)^2 \cdot X(j\Omega)$$

Passive Circuit: Frequency Response



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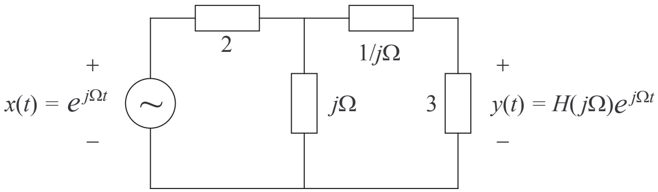
- By FT time-domain differentiation property:

Passive Circuit: Frequency Response



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- By FT time-domain differentiation property:

$$5\ddot{y}(t) + 7\dot{y}(t) + 2y(t) \;=\; 3\ddot{x}(t)$$

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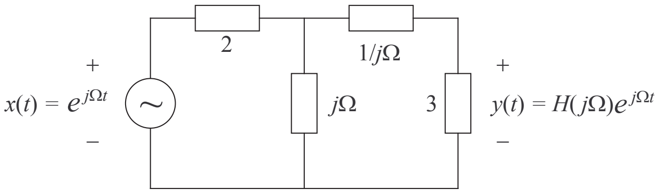

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- By FT time-domain differentiation property:

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  - $$h(t) \;=\; \frac{3}{5}\delta(t) - e^{-t} + \frac{4}{25}e^{-2t/5}$$