

Lecture 20

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- ▶ Amplitude modulation

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Fourier Series vs. DFT

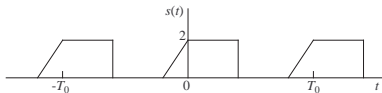
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- ▶ Time-frequency duality

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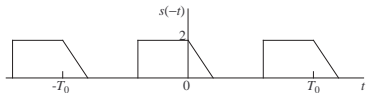
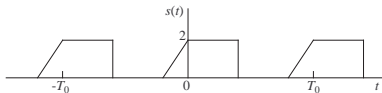
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- ▶ Time-frequency duality is more prominent in DFT than in Fourier series.

Even and Odd Symmetry

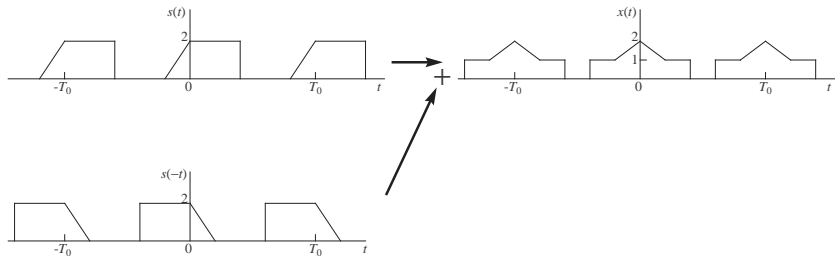
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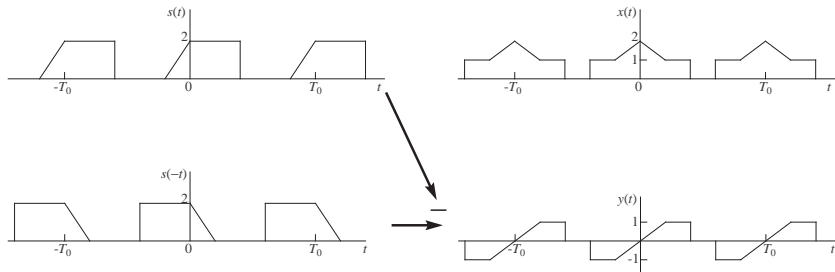
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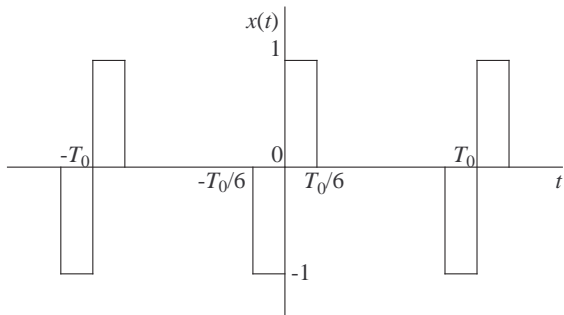


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Example: Time Delay

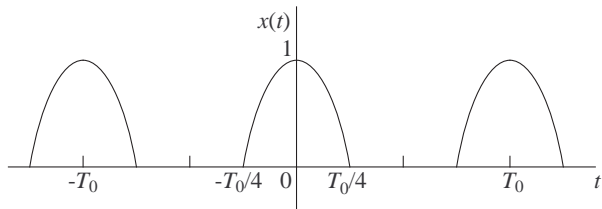
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Example: Modulation

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Half-wave rectification:



Half Wave: Fourier Series Coefficients

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