Lecture 19

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- Fourier series of a periodic signal:

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- Orthogonality of Fourier sinusoids
- Example: Fourier series for rectangular pulse train

Fourier Series: Synthesis and Analysis

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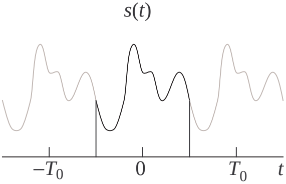
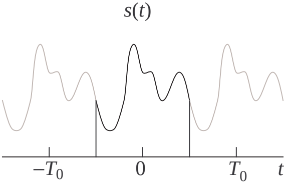


Fourier Series: Synthesis and Analysis



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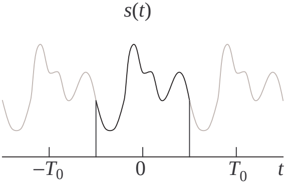

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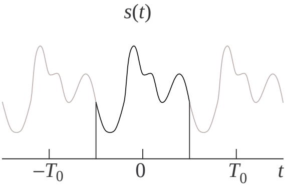

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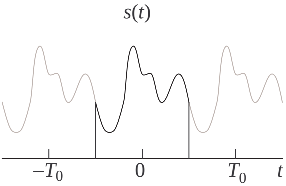

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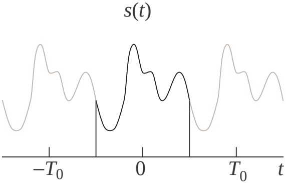

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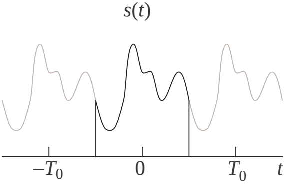

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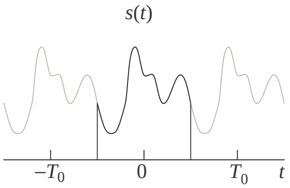

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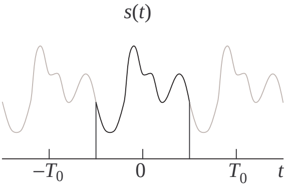

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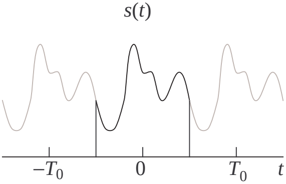

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- Key result:

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- Key result: Fourier sinusoids $v^{(k)}(t)$ are mutually orthogonal

Rectangular Pulse Train: FS Coefficients