Lecture 18

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- Bandlimited signals (finite Fourier series)

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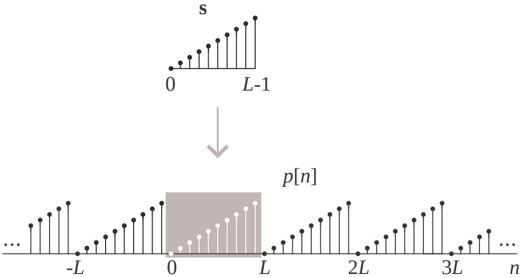

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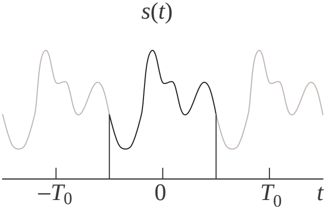

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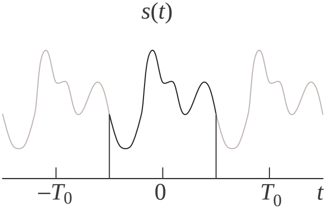

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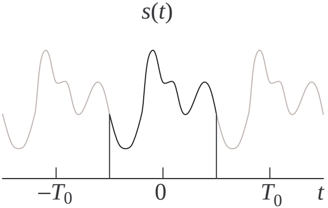

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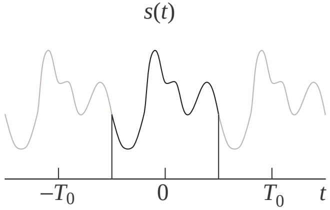

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- $v^{(k)}(t)$: complex sinusoid of frequency $k/T_0 = k f_0$ (Hz)

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