# Lecture 15

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- Derivation of new DFT pairs from $\mathbf{x} \longleftrightarrow \mathbf{X}$

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- Derivation of new DFT pairs from $\mathbf{x} \longleftrightarrow \mathbf{X}$
- Duality between circular shift $\mathbf{P}$ and modulation $\mathbf{F}$
- Duality between DFT and its inverse
- Symmetry properties of DFT

# Last Lecture

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- Circular Shift $\mathbf{P}$

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- *Time Domain* *Frequency Domain*

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$$\mathbf{x} = \frac{1}{N}\mathbf{V}\mathbf{X}$$

# Last Lecture

- Circular Shift $\mathbf{P}$
- Circular Reversal $\mathbf{R}$
- 

| *Time Domain* | | *Frequency Domain* |
|---|---|---|
| $\mathbf{x} = \frac{1}{N}\mathbf{V}\mathbf{X}$ | $\longleftrightarrow$ | $\mathbf{X} = \mathbf{V}^*\mathbf{x}$ |

# Last Lecture

- Circular Shift $\mathbf{P}$
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- | *Time Domain* | | *Frequency Domain* |
  | --- | --- | --- |
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- Circular Shift $\mathbf{P}$
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- *Time Domain* | *Frequency Domain*

$$\mathbf{x} = \frac{1}{N}\mathbf{V}\mathbf{X} \quad \longleftrightarrow \quad \mathbf{X} = \mathbf{V}^*\mathbf{x} = \mathbf{W}\mathbf{x}$$

$$\mathbf{R}\mathbf{x}$$

# Last Lecture

- Circular Shift $\mathbf{P}$
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- | *Time Domain* | | *Frequency Domain* |
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  | $\mathbf{x} = \frac{1}{N}\mathbf{V}\mathbf{X}$ | $\longleftrightarrow$ | $\mathbf{X} = \mathbf{V}^*\mathbf{x} = \mathbf{W}\mathbf{x}$ |
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- Circular Shift $\mathbf{P}$
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- 

| *Time Domain* | | *Frequency Domain* |
| --- | --- | --- |
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| $\mathbf{R}\mathbf{x}$ | $\longleftrightarrow$ | $\mathbf{R}\mathbf{X}$ |
| $\mathbf{x}^*$ | | |

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| *Time Domain* | | *Frequency Domain* |
|---|---|---|
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| $\mathbf{x}^*$ | $\longleftrightarrow$ | $\mathbf{R}\mathbf{X}^*$ |

# Last Lecture

- Circular Shift $\mathbf{P}$
- Circular Reversal $\mathbf{R}$
- *Time Domain* ⟷ *Frequency Domain*

$$\begin{aligned} \mathbf{x} = \frac{1}{N}\mathbf{V}\mathbf{X} \quad &\longleftrightarrow \quad \mathbf{X} = \mathbf{V}^*\mathbf{x} = \mathbf{W}\mathbf{x} \\ \mathbf{R}\mathbf{x} \quad &\longleftrightarrow \quad \mathbf{R}\mathbf{X} \\ \mathbf{x}^* \quad &\longleftrightarrow \quad \mathbf{R}\mathbf{X}^* \end{aligned}$$

- Diagonal modulation matrix $\mathbf{F}$

# Last Lecture

- Circular Shift $\mathbf{P}$
- Circular Reversal $\mathbf{R}$
-

| *Time Domain* | | *Frequency Domain* |
|---|---|---|
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- Diagonal modulation matrix $\mathbf{F}$ has $k = 1^{\mathrm{st}}$ Fourier sinusoid on the leading diagonal

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- Diagonal modulation matrix $\mathbf{F}$ has $k = 1^{\text{st}}$ Fourier sinusoid on the leading diagonal
- $\mathbf{F}^k\mathbf{x}$: entry-wise product of $\mathbf{x}$ and $k^{\text{th}}$ Fourier sinusoid

# Further DFT Pairs

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- Auxiliary identities:

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$$\mathbf{V}\mathbf{P}^m = \mathbf{F}^m\mathbf{V} \ , \qquad \mathbf{P}^m\mathbf{V} = \mathbf{V}\mathbf{F}^{-m}$$

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- *Time Domain* $\qquad$ *Frequency Domain*

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- 

| *Time Domain* | | *Frequency Domain* |
|---|---|---|
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| $\mathbf{P}^m\mathbf{x}$ | | |

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- 

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| $\mathbf{x} = \frac{1}{N}\mathbf{V}\mathbf{X}$ | $\longleftrightarrow$ | $\mathbf{X} = \mathbf{W}\mathbf{x}$ |
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DFT Duality:

# Further DFT Pairs

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DFT Duality: $\mathbf{X}$

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| $\mathbf{P}^m\mathbf{x}$ | $\longleftrightarrow$ | $\mathbf{F}^{-m}\mathbf{X}$ |
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DFT Duality: $\quad \mathbf{X} \longleftrightarrow N\mathbf{R}\mathbf{x}$

# Examples

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$$\begin{bmatrix} a & b & c & d & e & f \end{bmatrix}^T \longleftrightarrow \begin{bmatrix} A & B & C & D & E & F \end{bmatrix}^T$$

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$$\begin{bmatrix} d & e & f & a & b & c \end{bmatrix}^T \longleftrightarrow \begin{bmatrix} A & -B & C & -D & E & -F \end{bmatrix}^T$$

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$$\begin{bmatrix} 2a & b & -c & -2d & -e & f \end{bmatrix}^T \longleftrightarrow [\, F+B \;\; A+C \;\; B+D \;\; C+E \;\; D+F \;\; E+A \,]^T$$

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$$\begin{bmatrix} A & B & C & D & E & F \end{bmatrix}^T \longleftrightarrow 6\begin{bmatrix} a & f & e & d & c & b \end{bmatrix}^T$$

# Symmetry Properties

# Symmetry Properties

- $$\mathbf{x} \quad \longleftrightarrow \quad \mathbf{X}$$

# Symmetry Properties

- $$\begin{aligned} \mathbf{x} &\longleftrightarrow \mathbf{X} \\ \mathbf{Rx} &\longleftrightarrow \mathbf{RX} \end{aligned}$$

# Symmetry Properties

- $$\begin{aligned} \mathbf{x} &\longleftrightarrow \mathbf{X} \\ \mathbf{R}\mathbf{x} &\longleftrightarrow \mathbf{R}\mathbf{X} \\ \mathbf{x}^* &\longleftrightarrow \mathbf{R}\mathbf{X}^* \end{aligned}$$

# Symmetry Properties

- $$\begin{aligned} \mathbf{x} &\longleftrightarrow \mathbf{X} \\ \mathbf{R}\mathbf{x} &\longleftrightarrow \mathbf{R}\mathbf{X} \\ \mathbf{x}^* &\longleftrightarrow \mathbf{R}\mathbf{X}^* \end{aligned}$$
- real-valued $\longleftrightarrow$ circ. conjugate symmetric

# Symmetry Properties

- $$\begin{aligned} \mathbf{x} &\longleftrightarrow \mathbf{X} \\ \mathbf{Rx} &\longleftrightarrow \mathbf{RX} \\ \mathbf{x}^* &\longleftrightarrow \mathbf{RX}^* \end{aligned}$$

- $$\begin{aligned} \text{real-valued} &\longleftrightarrow \text{circ. conjugate symmetric} \\ \text{circ. conjugate symmetric} &\longleftrightarrow \text{real-valued} \end{aligned}$$

# Symmetry Properties

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- | | | |
  |---|---|---|
  | real-valued | $\longleftrightarrow$ | circ. conjugate symmetric |
  | circ. conjugate symmetric | $\longleftrightarrow$ | real-valued |
  | real and circ. symmetric | $\longleftrightarrow$ | real and circ. symmetric |

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- Real values preserved by:

  $$\mathbf{R}$$

# Symmetry Properties

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- Real values preserved by:

  $$\mathbf{R}, \qquad \mathbf{P}^m$$

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- Real values preserved by:

$$\mathbf{R}, \qquad \mathbf{P}^m \qquad \text{and} \qquad \mathbf{F}^m + \mathbf{F}^{-m}$$

# Symmetry Properties

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# Symmetry Properties

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