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Columns of $\mathbf{V}$ are orthogonal, each with $(\text{norm})^2 \; = \; 1$

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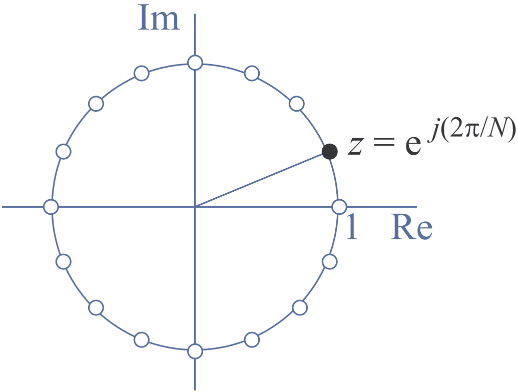
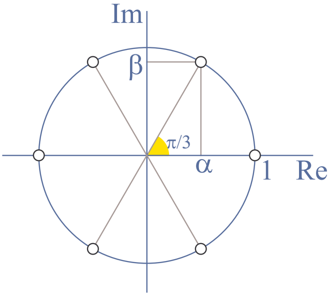
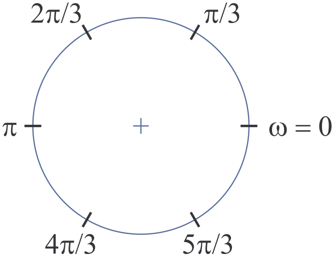

Im
$z = e^{j(2\pi/N)}$
1
Re


Im
β
π/3
α
1
Re


2π/3
π/3
π
ω = 0
4π/3
5π/3