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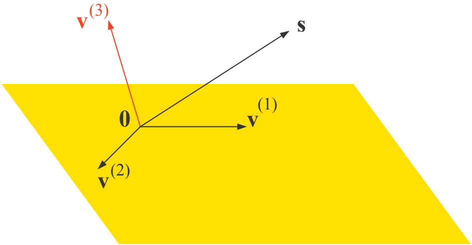
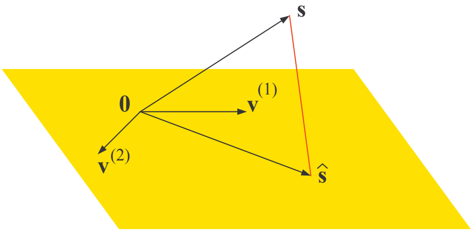
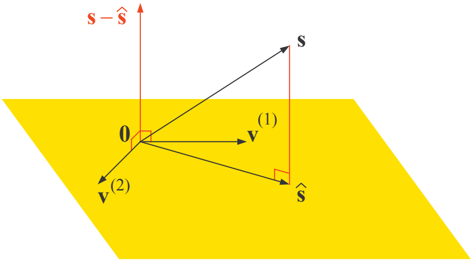
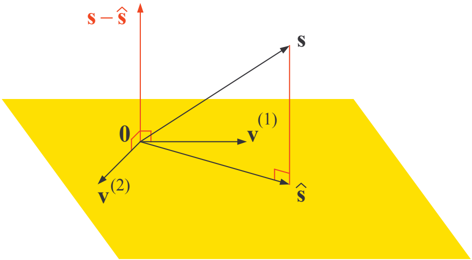


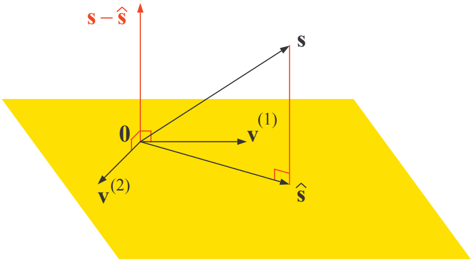
v(3)
s
0
v(1)
v(2)


s
0
v(1)
v(2)
ŝ


$\mathbf{s} - \hat{\mathbf{s}}$
$\mathbf{s}$
$\mathbf{0}$
$\mathbf{v}^{(1)}$
$\mathbf{v}^{(2)}$
$\hat{\mathbf{s}}$



Error norm $\|\mathbf{s} - \hat{\mathbf{s}}\|$



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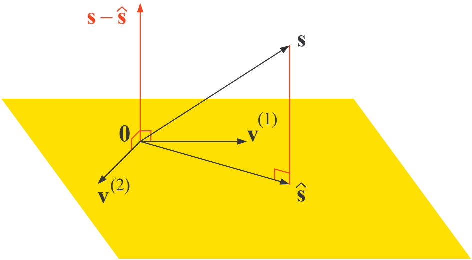

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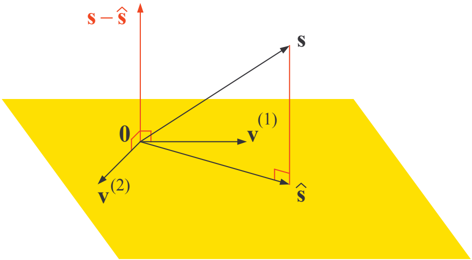

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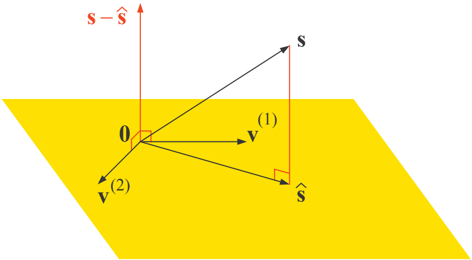

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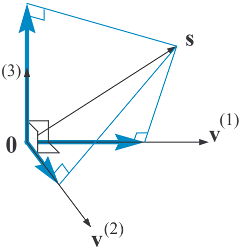

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