Lecture 11

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- Signal representation and approximation using orthogonal reference vectors

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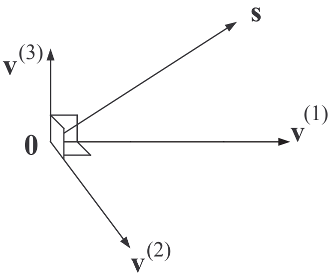
- Inner products and projections: extension to complex vectors

Signal Representation/Approximation



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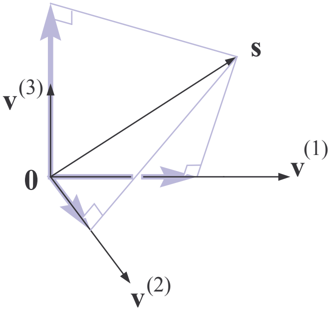

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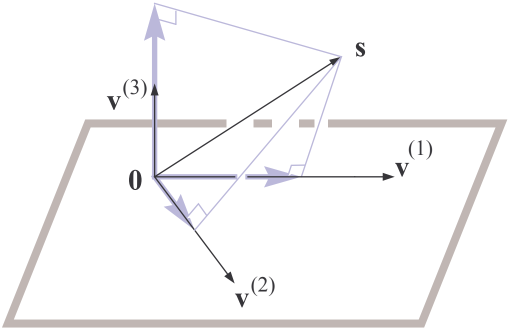

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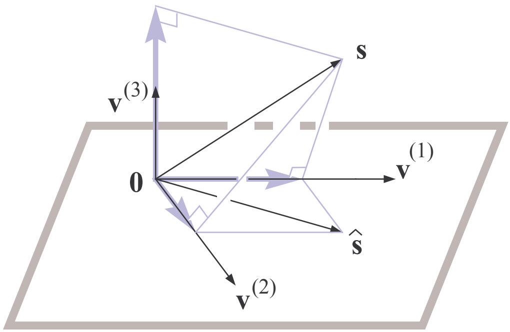

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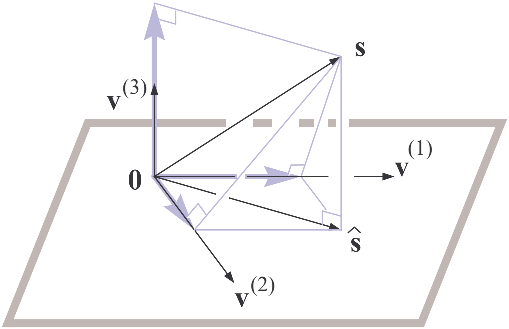

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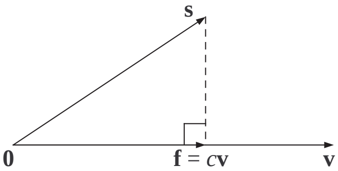

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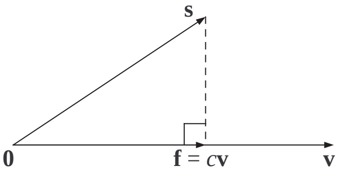

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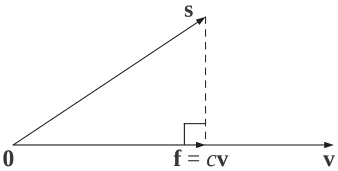

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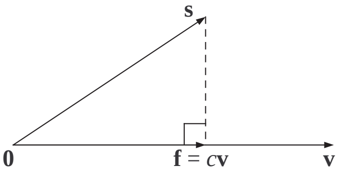

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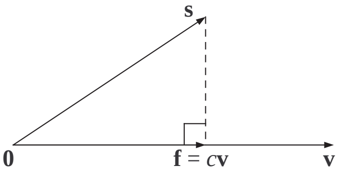

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$$\langle \mathbf{v},\, \mathbf{s} - c\mathbf{v} \rangle \;=\; 0 \qquad \Leftrightarrow \qquad c \;=\; \frac{\langle \mathbf{v}, \mathbf{s} \rangle}{\|\mathbf{v}\|^2}$$