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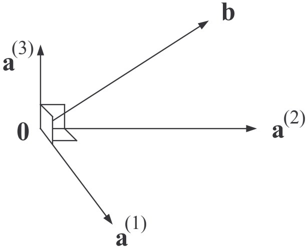

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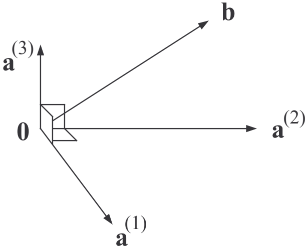

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$$(\forall k) \qquad c_k = \frac{\langle \mathbf{v}^{(k)}, \mathbf{s} \rangle}{\|\mathbf{v}^{(k)}\|^2}$$