Lecture 9

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- Some issues in solving $\mathbf{Ax} = \mathbf{b}$
- Nonsingularity and the matrix inverse $\mathbf{A}^{-1}$
- Properties of $\mathbf{A}^{-1}$
- Inversion of a triangular matrix
- Solution of $\mathbf{Ax} = \mathbf{b}$ by Gaussian elimination

Solving $\mathbf{Ax} = \mathbf{b}$

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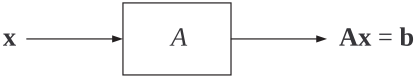
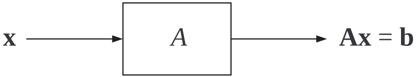


- Problem: determine the input $\mathbf{x}$ of a linear transformation (or system) based on the observed output $\mathbf{y} = \mathbf{b}$

Solving $\mathbf{Ax} = \mathbf{b}$



- Problem: determine the input $\mathbf{x}$ of a linear transformation (or system) based on the observed output $\mathbf{y} = \mathbf{b}$

- The dimensions of the input (n) and output (m) play a crucial role here.

The Case $m < n$

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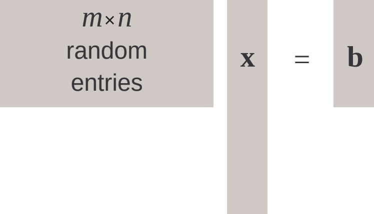

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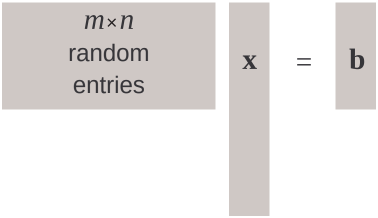

- Every $\mathbf{b}$ is almost certainly a valid system output, thus a solution $\mathbf{x}$ exists.

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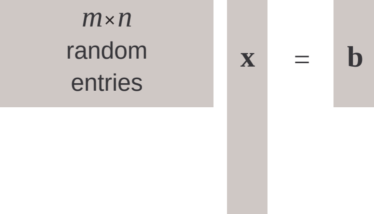

- Every $\mathbf{b}$ is almost certainly a valid system output, thus a solution $\mathbf{x}$ exists.
- The solution is *not* unique, thus the true input cannot be determined.

The Case $m > n$

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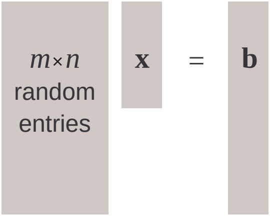

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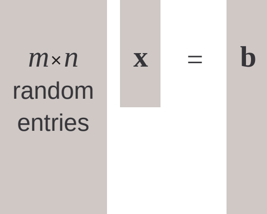
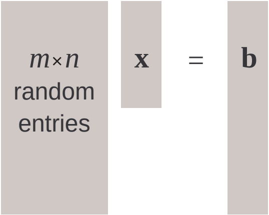


- A *random* $\mathbf{b}$ is almost certainly *not* a valid system output, thus a solution does not exist.

The Case $m > n$



- A *random* $\mathbf{b}$ is almost certainly *not* a valid system output, thus a solution does not exist.
- If $\mathbf{b}$ is a valid system output, a solution $\mathbf{x}$ exists and is almost certainly unique.

The Case $m = n$

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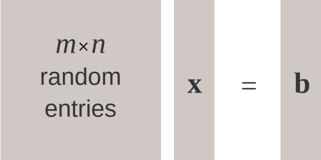

- Every $\mathbf{b}$ is almost certainly a valid system output corresponding to a unique system input $\mathbf{x}$.

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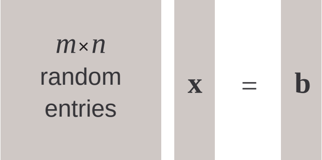

- Every $\mathbf{b}$ is almost certainly a valid system output corresponding to a unique system input $\mathbf{x}$.
- In other words, a solution $\mathbf{x}$ exists for every $\mathbf{b}$, and is unique.

Properties of $\mathbf{A}^{-1}$

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- $\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$

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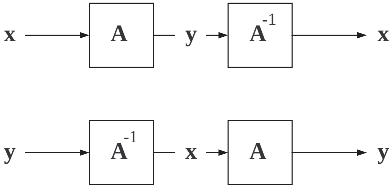

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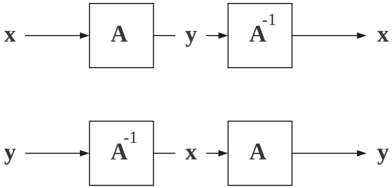

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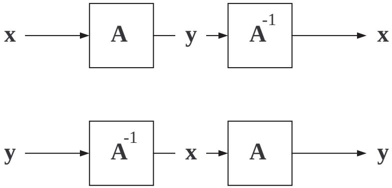

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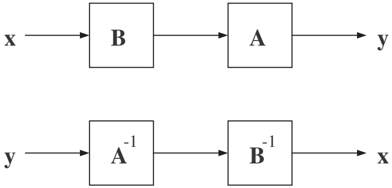

Example: Gaussian Elimination

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$$\begin{aligned}
2x_1 + x_2 - x_3 &= 6 \\
4x_1 - x_3 &= 6 \\
-8x_1 + 2x_2 + 3x_3 &= -10
\end{aligned}$$

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$$\begin{aligned} 2x_1 + x_2 - x_3 &= 6 \\ 4x_1 - x_3 &= 6 \\ -8x_1 + 2x_2 + 3x_3 &= -10 \end{aligned}$$