Lecture 7

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- Linear transformations of vectors

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- Matrix of a linear transformation

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- Linear transformation as a matrix-vector product

Matrix and Vector Notation

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$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in \mathbf{R}^{n\times 1}; \quad \mathbf{a}^T = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix} \in \mathbf{R}^{1\times n}$$

Linear Transformation $A : \mathbf{R}^n \to \mathbf{R}^m$

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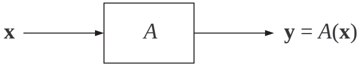

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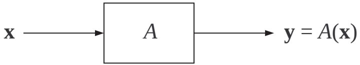
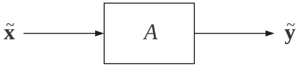


If also



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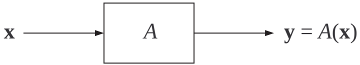

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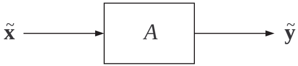
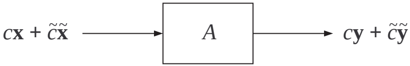


then



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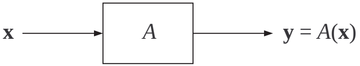

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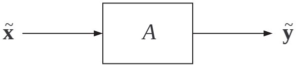

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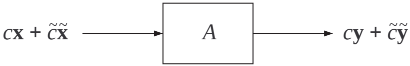

Definition of linearity:

$$A(c\mathbf{x} + \tilde{c}\tilde{\mathbf{x}}) \;=\; cA(\mathbf{x}) + \tilde{c}A(\tilde{\mathbf{x}})$$

Examples of Linear Transformations

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- Permutation

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- Projection

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- Permutation
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- Rotation