4×4 Matrix of Fourier (DFT) Sinsoids

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$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \quad \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} \Bigg\} \text{ Time } n$$

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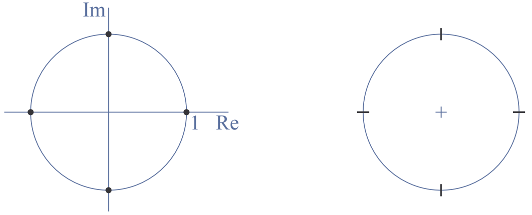

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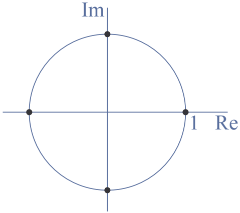

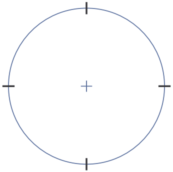

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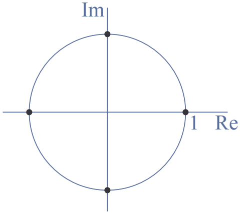

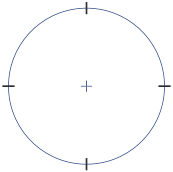

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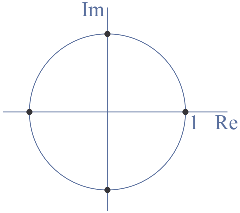

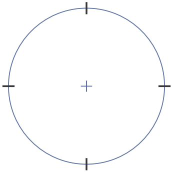

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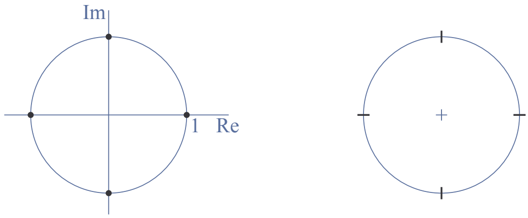

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$$\omega_k = k \cdot \pi/2$$

The $N \times N$ Matrix $\mathbf{V}$

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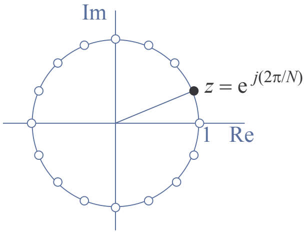


1. What is the smallest vector length N for which both

$$\omega = \frac{7\pi}{12} \qquad \text{and} \qquad \omega' = \frac{11\pi}{28}$$

are Fourier (DFT) frequencies?

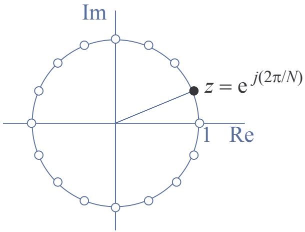
A. 77

B. 84

C. 168

D. 336

The $N \times N$ Matrix $\mathbf{V}$



$$V_{nk} = v^{(k)}[n] = z^{kn}$$

2. What is the smallest vector length N such that the $N \times N$ matrix $\mathbf{V}$ of Fourier sinusoids contains the entry

$$-\frac{\sqrt{3}}{2} + \frac{j}{2} \; ?$$

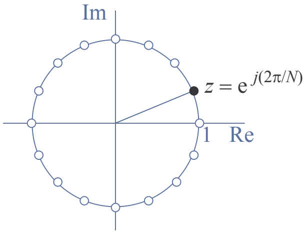
A. 6

B. 8

C. 12

D. 24

The $N \times N$ Matrix $\mathbf{V}$



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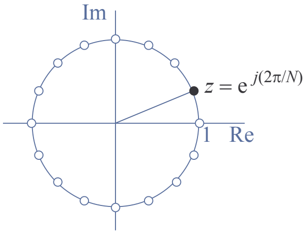

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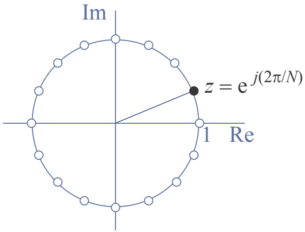

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$$\mathbf{V} = \begin{bmatrix} 1 & & & & \\ 1 & & & & \\ 1 & & & & \\ \vdots & & & & \\ 1 & & & & \end{bmatrix} = \mathbf{V}^T$$

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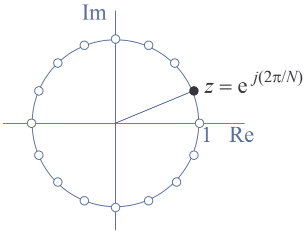

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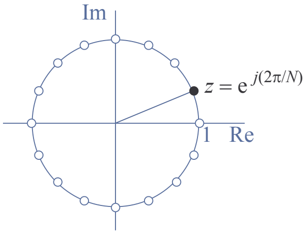

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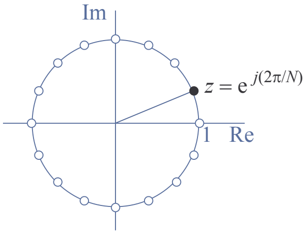

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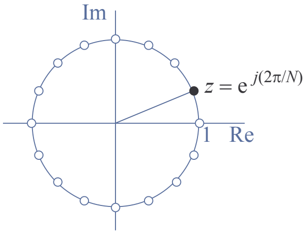

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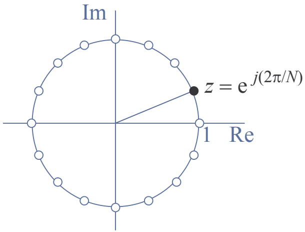

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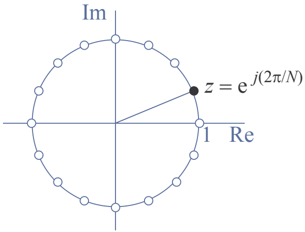

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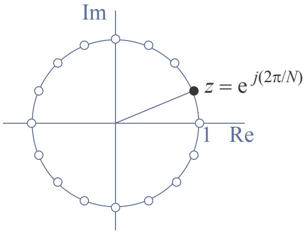

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The Discrete Fourier Transform

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3. If $\mathbf{s} = [\ 7 \quad -2 \quad 3 \quad -1 \quad 4 \quad 5\ ]^T$ has DFT $S[0:5]$, then $S[3]$ equals

A. 12

B. 16

C. $1 + j(3\sqrt{3})$

D. $1 - j(3\sqrt{3})$

4. Let $\mathbf{x} = x_{0:7}$. Which of the following entries in the DFT $\mathbf{X}$ is given by

$$x_0 + jx_1 - x_2 - jx_3 + x_4 + jx_5 - x_6 - jx_7 \text{ ?}$$

A. $X[1]$

B. $X[2]$

C. $X[6]$

D. $X[7]$

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5. The signal vector $\mathbf{x} = x[0:15]$ is given by the formula

$$x[n] \;=\; \cos(5\pi n/8), \qquad n = 0, \ldots, 15$$

How many zeros does the DFT vector $\mathbf{X}$ contain?

A. None

B. One

C. Fourteeen

D. Fifteen

6. Which of the following statements are true about the DFT $\mathbf{X}$ of a signal vector $\mathbf{x}$?

A. $\mathbf{x}$ and $\mathbf{X}$ are vectors of the same length.

B. $\mathbf{X}$ contains information about the amplitude and phase of standard sinusoidal vectors which, when summed together, produce the signal vector $\mathbf{x}$.

C. If one of the entries of the DFT $\mathbf{X}$ is zero, then $\mathbf{x}$ must also contain (at least) one zero entry.

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