The Matrix Product $\mathbf{AB}$

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Defined as the matrix of the cascade

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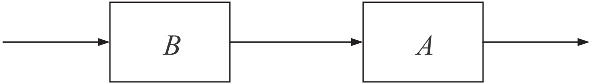

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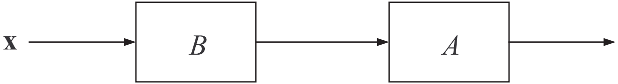

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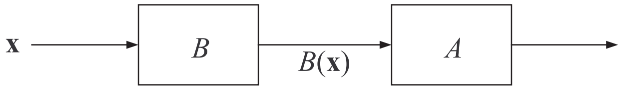

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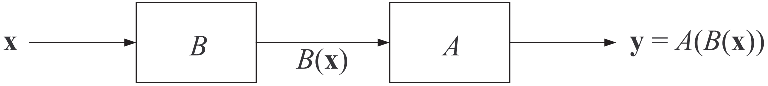

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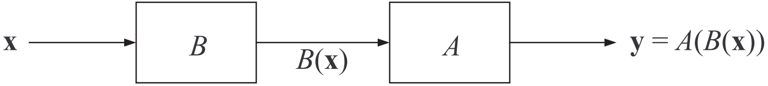

$$(\mathbf{AB})\mathbf{x}$$

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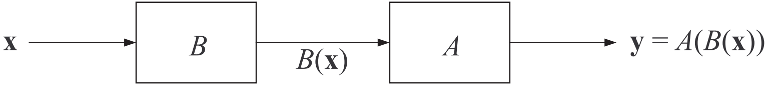

$$(\mathbf{AB})\mathbf{x} \;=\; A(B(\mathbf{x}))$$

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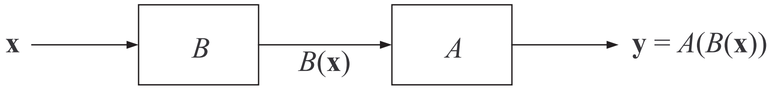

$$(\mathbf{AB})\mathbf{x} = A(B(\mathbf{x})) = \mathbf{A}(\mathbf{B}\mathbf{x})$$

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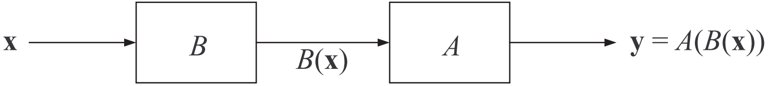

$$(\mathbf{AB})\mathbf{x} \;=\; A(B(\mathbf{x})) \;=\; \mathbf{A}(\mathbf{Bx})$$

$$(\mathbf{AB})_{ij} \;=$$

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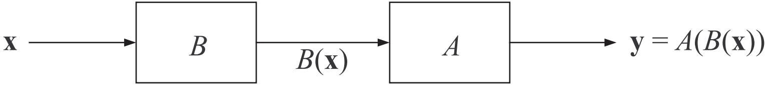

$$(\mathbf{AB})\mathbf{x} \;=\; A(B(\mathbf{x})) \;=\; \mathbf{A}(\mathbf{B}\mathbf{x})$$

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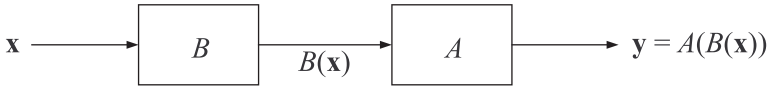

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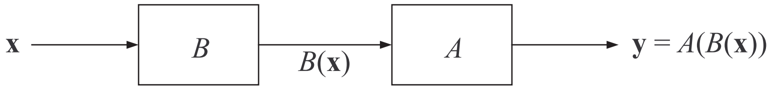

$$(\mathbf{AB})\mathbf{x} \;=\; A(B(\mathbf{x})) \;=\; \mathbf{A}(\mathbf{Bx})$$

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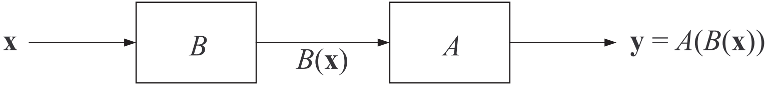

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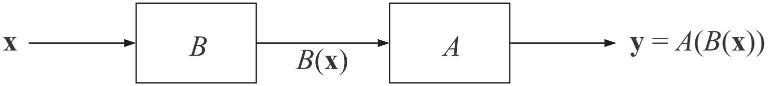

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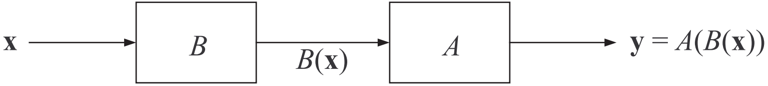

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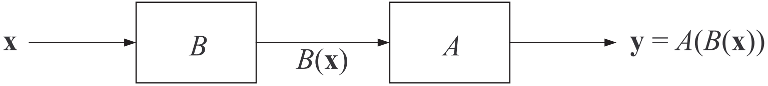

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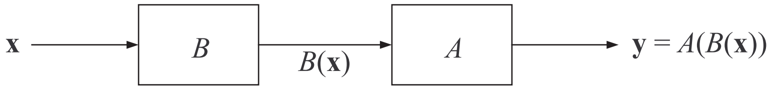

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$$\mathbf{A}: \; m \times p, \;\; \mathbf{B}: \; p \times n \quad \Rightarrow \quad \mathbf{AB}: \; m \times n$$

1. If

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 4 \\ 2 & 0 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 5 & -1 \\ 1 & 2 \\ -1 & 3 \end{bmatrix},$$

then $\mathbf{AB} =$

Associativity and Non-Commutativity

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Matrix product is associative:

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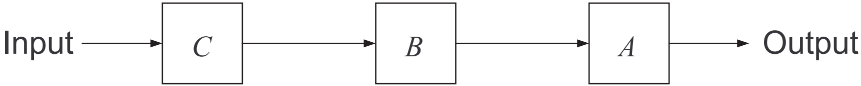

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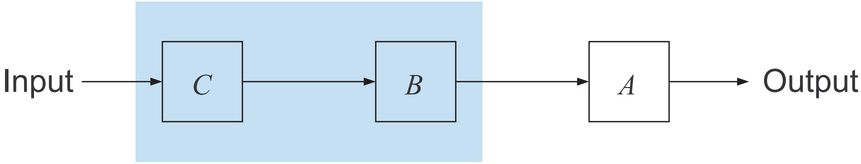

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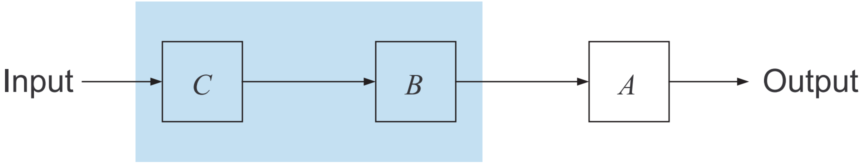

$$\mathbf{A}(\mathbf{BC})$$

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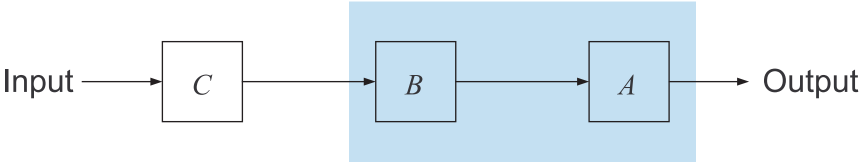

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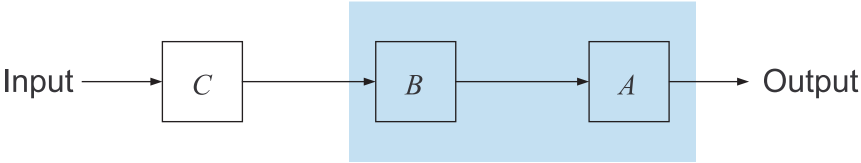

$$\mathbf{A}(\mathbf{B}\mathbf{C}) = (\mathbf{A}\mathbf{B})\mathbf{C}$$

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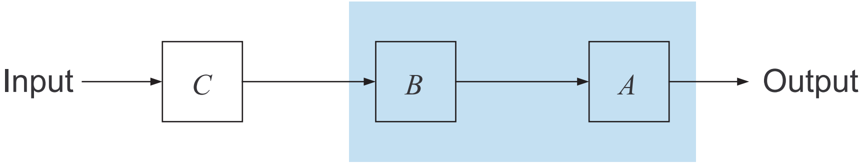

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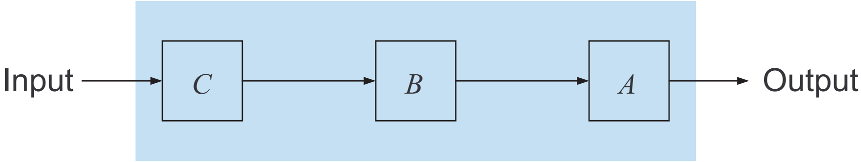

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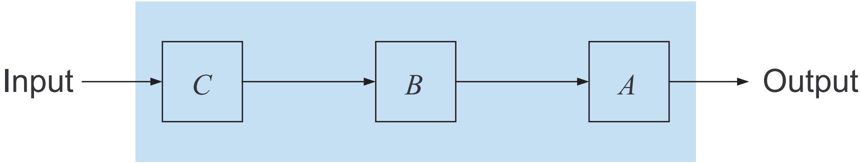

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Matrix product is not commutative:

Associativity and Non-Commutativity

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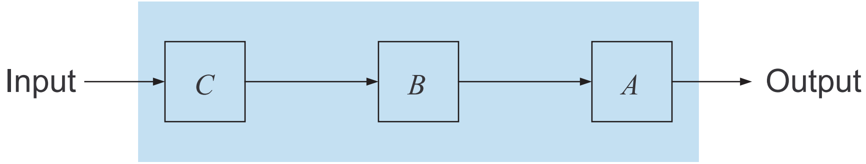


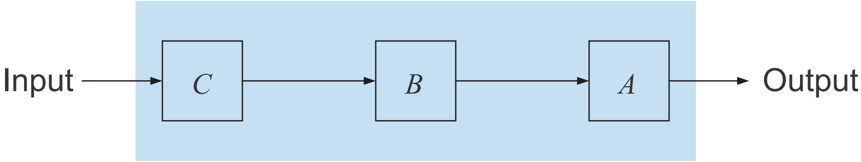
$$\mathbf{A}(\mathbf{B}\mathbf{C}) = (\mathbf{A}\mathbf{B})\mathbf{C} = \mathbf{A}\mathbf{B}\mathbf{C}$$

Matrix product is not commutative:

$$\mathbf{A}\mathbf{B} \neq \mathbf{B}\mathbf{A}$$

Associativity and Non-Commutativity

Matrix product is associative:



$$\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C} = \mathbf{ABC}$$

Matrix product is not commutative:

$$\mathbf{AB} \neq \mathbf{BA} \quad \text{(in general)}$$

2. In which (one or more) of the following instances does

$$\mathbf{AB} = \mathbf{BA}$$

hold?

(i) $\mathbf{A}$ and $\mathbf{B}$ are square matrices of the same size.

(ii) $\mathbf{A}$ and $\mathbf{B}$ are diagonal matrices of the same size.

(iii) $\mathbf{A}$ is a 2×2 matrix representing a counterclockwise rotation by $\pi/6$ on the Cartesian plane; while $\mathbf{B}$ is a 2×2 matrix representing projection onto the horizontal axis (of the same plane).

(iv) $\mathbf{A}$ is a square matrix and $\mathbf{B}$ is the identity matrix (of the same size).

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3. Let

$$\mathbf{A} = \begin{bmatrix} \cos(\pi/8) & -\sin(\pi/8) \\ \sin(\pi/8) & \cos(\pi/8) \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \cos(3\pi/7) & \sin(3\pi/7) \\ -\sin(3\pi/7) & \cos(3\pi/7) \end{bmatrix}$$

Which (only one) of the following exponent pairs (m, n) is such that

$$\mathbf{A}^m \mathbf{B}^n = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} ?$$

(i) $(8, 7)$ (ii) $(8, 14)$ (iii) $(4, 7)$ (iv) $(4, 14)$

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4. Let $\mathbf{a}$ and $\mathbf{b}$ be n-dimensional column vectors (where $n > 1$) having real-valued entries. If

$$\mathbf{C} = \mathbf{a}^T\mathbf{b}\,\mathbf{b}^T\mathbf{a} \;,$$

which (one ore more) of the following statements are true about $\mathbf{C}$?

(i) $\mathbf{C}$ is a $n \times n$ matrix.

(ii) $\mathbf{C}$ is scalar (i.e., 1×1).

(iii) $\mathbf{C} = \mathbf{C}^T$

(iv) The entries of $\mathbf{C}$ may be positive, zero or negative, depending on the choice of $\mathbf{a}$ and $\mathbf{b}$.

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Column and Row Operations

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- $\mathbf{Ac} =$

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If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac} =$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac} \;=\;$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] \;=$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac} =$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] = [\,\mathbf{Ac} \mid \mathbf{Ad}\,]$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac}$ $=$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] = [\,\mathbf{Ac} \mid \mathbf{Ad}\,] =$ two such linear combinations (side by side)

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If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

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- $\mathbf{Ae}^{(j)} =$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac} \;=\;$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] \;=\; [\,\mathbf{Ac} \mid \mathbf{Ad}\,] \;=\;$ two such linear combinations (side by side)
- $\mathbf{Ae}^{(j)} = \; j^{\text{th}}$ column of $\mathbf{A}$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{A}\mathbf{c} \;=\;$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] \;=\; [\,\mathbf{A}\mathbf{c} \mid \mathbf{A}\mathbf{d}\,] \;=\;$ two such linear combinations (side by side)
- $\mathbf{A}\mathbf{e}^{(j)} =\; j^{\text{th}}$ column of $\mathbf{A}$

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Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac}$ = linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] = [\,\mathbf{Ac} \mid \mathbf{Ad}\,]$ = two such linear combinations (side by side)
- $\mathbf{Ae}^{(j)} =$ j^{th} column of $\mathbf{A}$

If $\mathbf{c}$ is $m \times 1$, then

- $\mathbf{c}^T\mathbf{A} =$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac} \;=\;$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] \;=\; [\,\mathbf{Ac} \mid \mathbf{Ad}\,] \;=\;$ two such linear combinations (side by side)
- $\mathbf{Ae}^{(j)} = \; j^{\text{th}}$ column of $\mathbf{A}$

If $\mathbf{c}$ is $m \times 1$, then

- $\mathbf{c}^T\mathbf{A} \;=\;$ linear combination of rows of $\mathbf{A}$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac}$ = linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] = [\,\mathbf{Ac} \mid \mathbf{Ad}\,]$ = two such linear combinations (side by side)
- $\mathbf{Ae}^{(j)} =$ j^{th} column of $\mathbf{A}$

If $\mathbf{c}$ is $m \times 1$, then

- $\mathbf{c}^T\mathbf{A}$ = linear combination of rows of $\mathbf{A}$
- $[\,\mathbf{c}^T \,;\, \mathbf{d}^T\,]\mathbf{A} =$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac} \;=\;$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] \;=\; [\,\mathbf{Ac} \mid \mathbf{Ad}\,] \;=\;$ two such linear combinations (side by side)
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If $\mathbf{c}$ is $m \times 1$, then

- $\mathbf{c}^T\mathbf{A} \;=\;$ linear combination of rows of $\mathbf{A}$
- $[\,\mathbf{c}^T \,;\, \mathbf{d}^T\,]\mathbf{A} \;=\; [\,\mathbf{c}^T\mathbf{A} \,;\, \mathbf{d}^T\mathbf{A}\,]$

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{Ac} \;=\;$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] \;=\; [\,\mathbf{Ac} \mid \mathbf{Ad}\,] \;=\;$ two such linear combinations (side by side)
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If $\mathbf{c}$ is $m \times 1$, then

- $\mathbf{c}^T\mathbf{A} \;=\;$ linear combination of rows of $\mathbf{A}$
- $[\,\mathbf{c}^T\,;\,\mathbf{d}^T\,]\mathbf{A} \;=\; [\,\mathbf{c}^T\mathbf{A}\,;\,\mathbf{d}^T\mathbf{A}\,] \;=\;$ two such linear combinations (stacked)

Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

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Column and Row Operations

If $\mathbf{A}$ is $m \times n$ and $\mathbf{c}$ is $n \times 1$, then

- $\mathbf{A}\mathbf{c} \;=\;$ linear combination of columns of $\mathbf{A}$ with coefficients in $\mathbf{c}$
- $\mathbf{A}[\,\mathbf{c} \mid \mathbf{d}\,] \;=\; [\,\mathbf{A}\mathbf{c} \mid \mathbf{A}\mathbf{d}\,] \;=\;$ two such linear combinations (side by side)
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$\mathbf{I} \;=\; (n \times n)$ identity matrix

5. Let

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ p & q & r \\ s & t & u \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{bmatrix} s & t & u \\ s & t & u \\ p & q & r \\ d & e & f \end{bmatrix}$$

Then $\mathbf{B} = \mathbf{PA}$, where $\mathbf{P} =$

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$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}, \qquad \mathbf{P} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \mathbf{AP}$$

Then $\mathbf{A} = \mathbf{BQ}$, where $\mathbf{Q} =$