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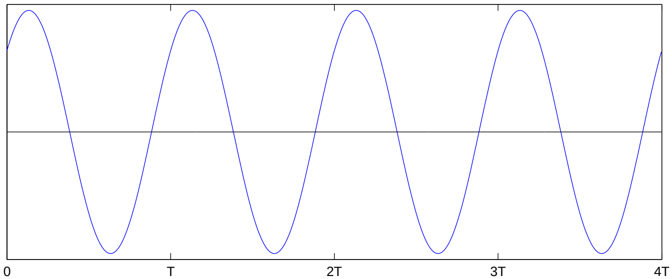

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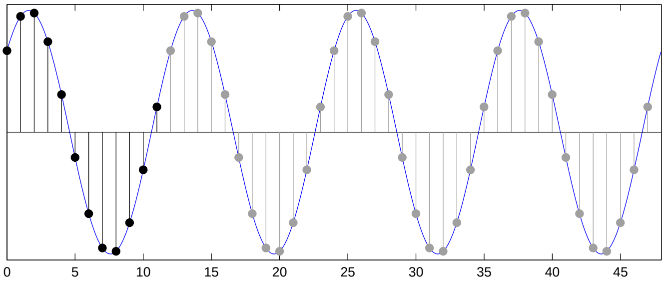

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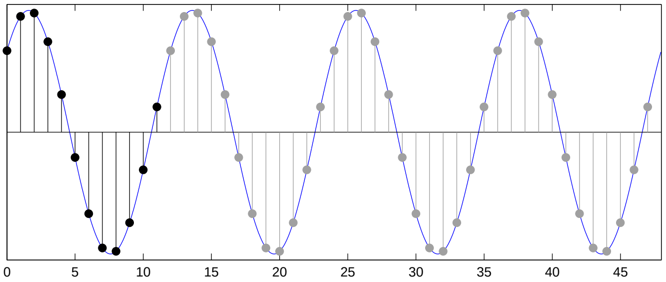


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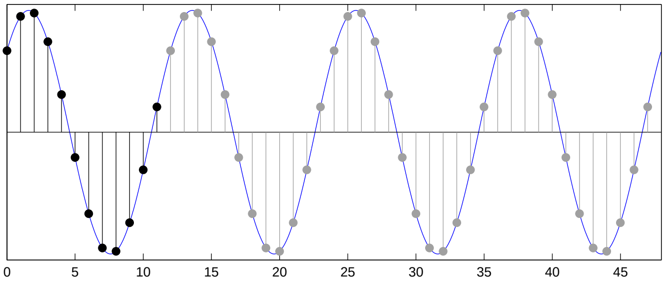


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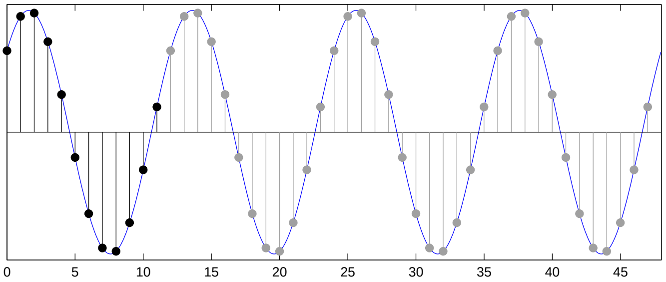


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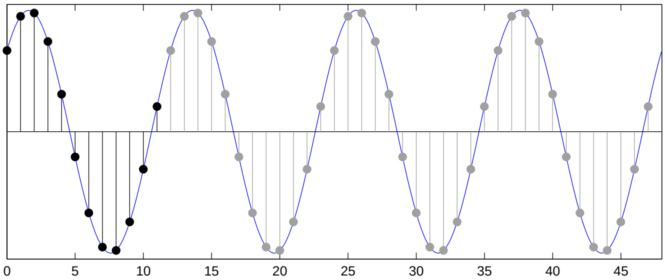


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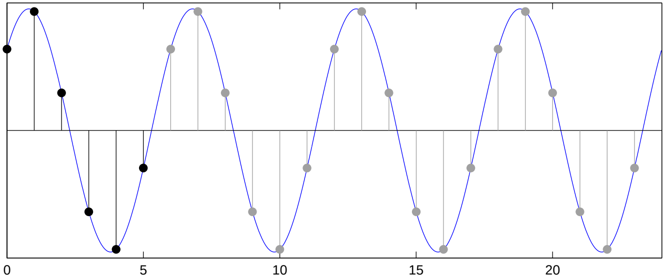


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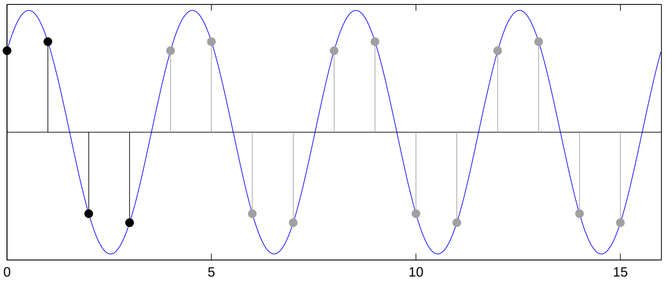


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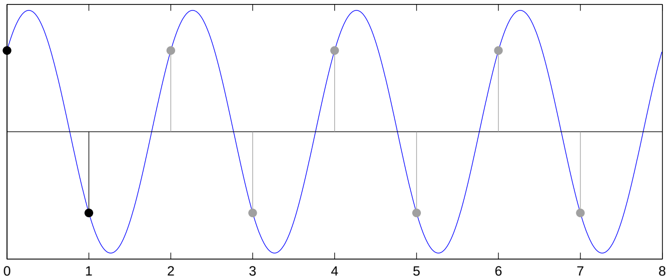


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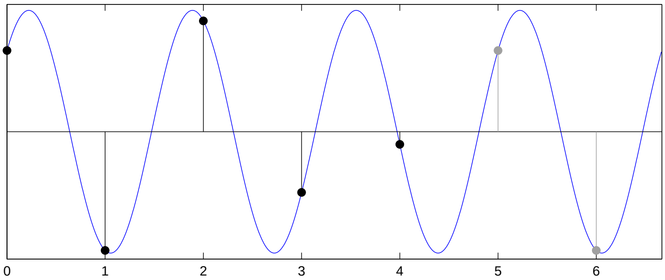


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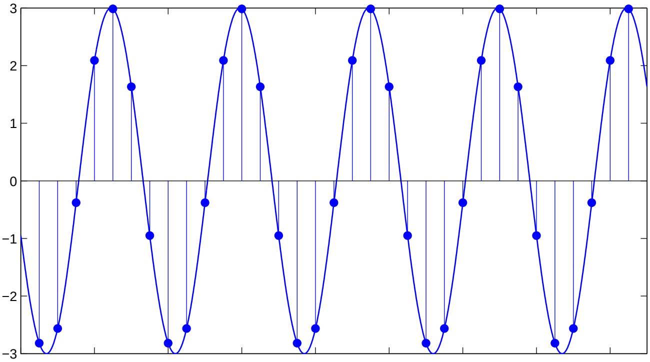
**3 and 4 (previous worksheet)** $x(t) = 3\cos(\Omega t + \phi)$ is plotted below (in black). The stem plot (in blue) is the sequence of samples $x[n] = x(nT_s)$.



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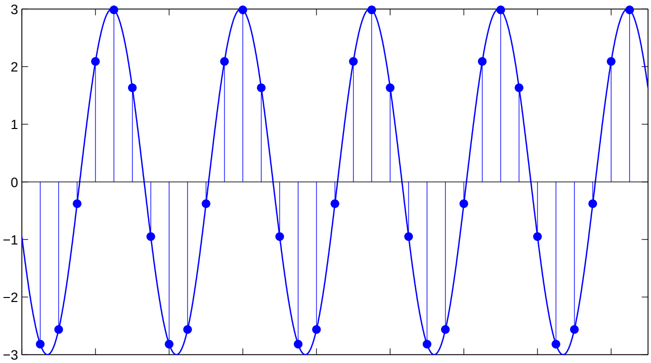


Write an equation for $x[n]$ that contains neither $\Omega$ nor $T_s$.

**5 (previous worksheet)** Let

$$x(t) \;=\; \cos(200\pi t + 1.7) \;,$$

where $t$ is in seconds. For which (one or more) of the following values of $T_s$ is the sample sequence $x[n] = x(nT_s)$ given by the equation

$$x[n] \;=\; \cos(0.7\pi n - 1.7) \;?$$

A. $3.5$ ms

B. $6.5$ ms

C. $13.5$ ms

D. $16.5$ ms

# Sampling and Interpolation (Analog $\rightleftarrows$ Digital)

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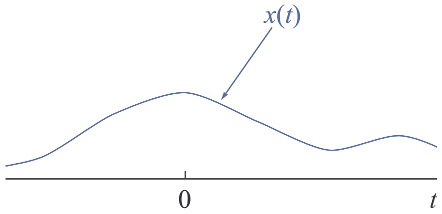

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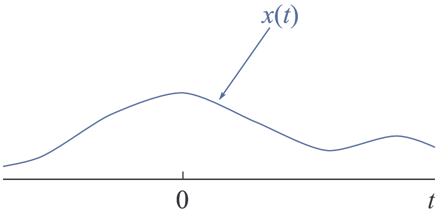


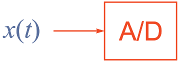

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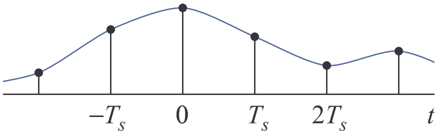


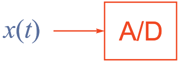

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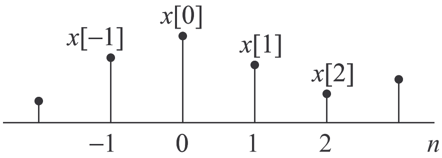


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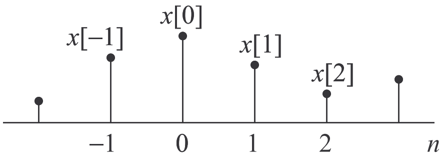


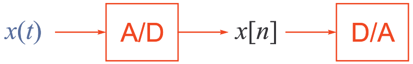

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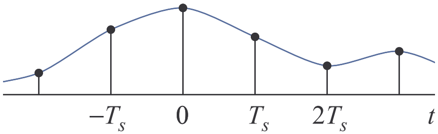


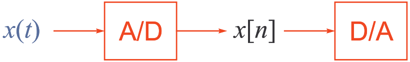

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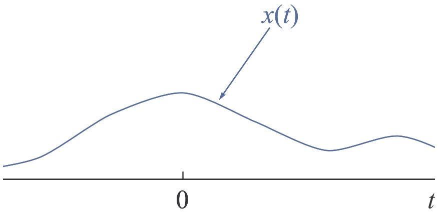


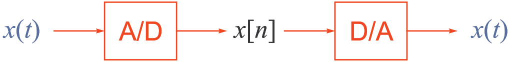

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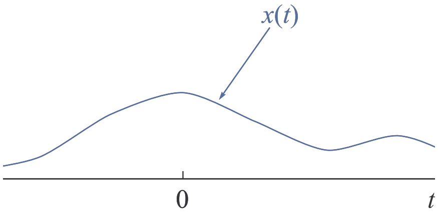


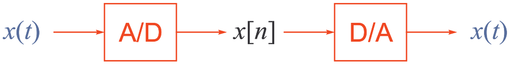


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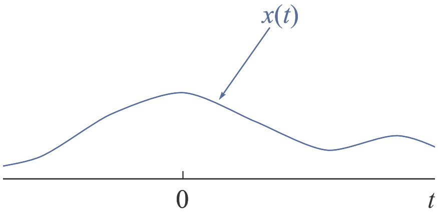


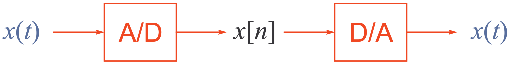


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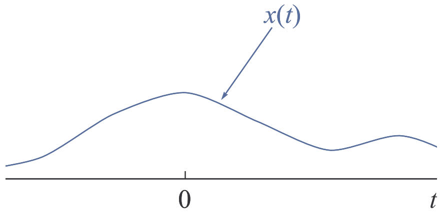


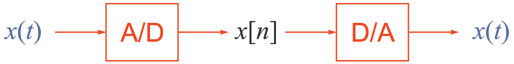


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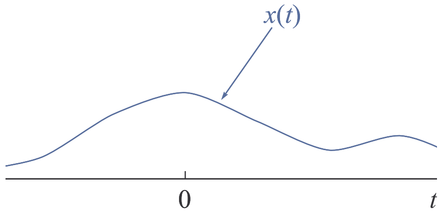


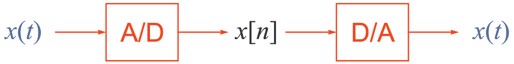


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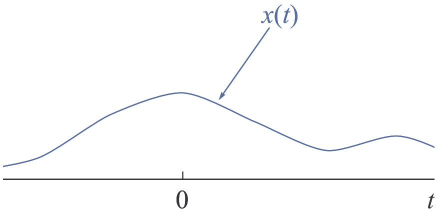


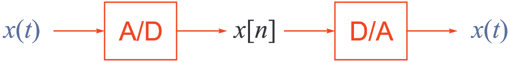


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In other words,

$$f' = \pm f + k f_s$$

(The two analog frequencies $f$ and $f'$ are indistinguishable based on the samples obtained.)

1. Which (one or more) of the following frequencies (in Hz) are aliases of $f = 70$ Hz when the sampling rate equals $f_s = 500$ samples per second?

A. 430

B. 570

C. 630

D. 770

# Aliases: Graphical Illustration

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Mapping $f \to \omega$ : $\omega = 2\pi(f/f_s)$

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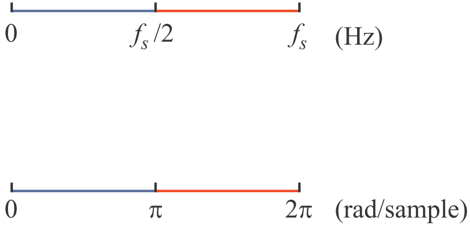

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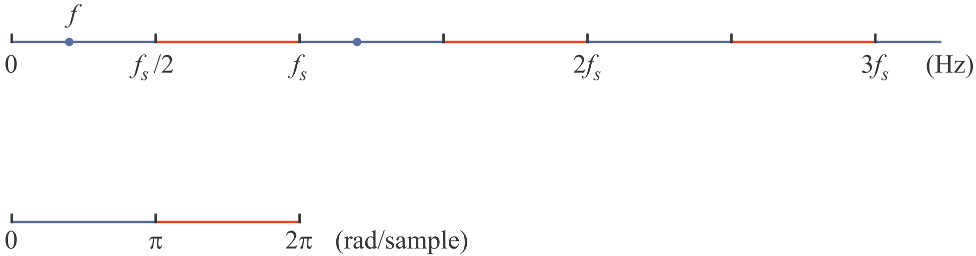

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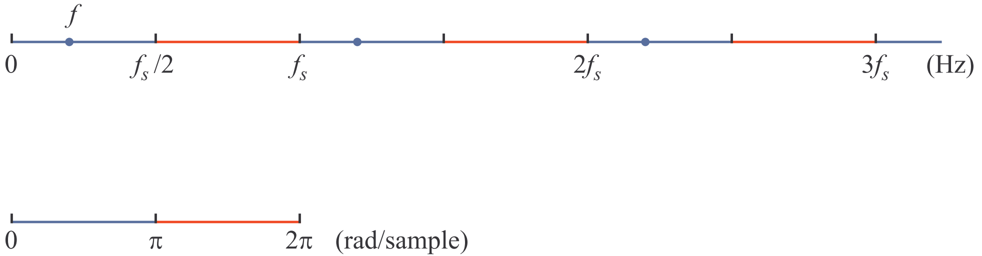

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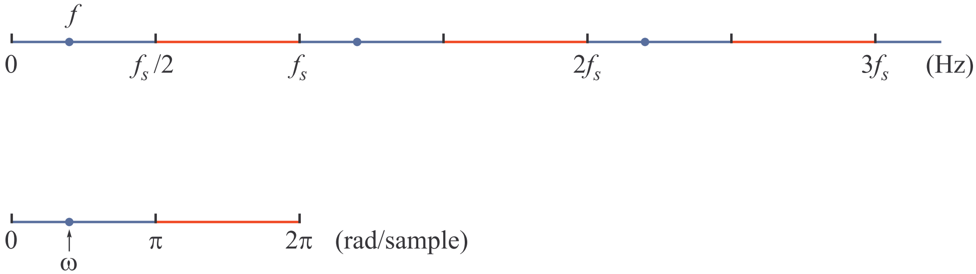

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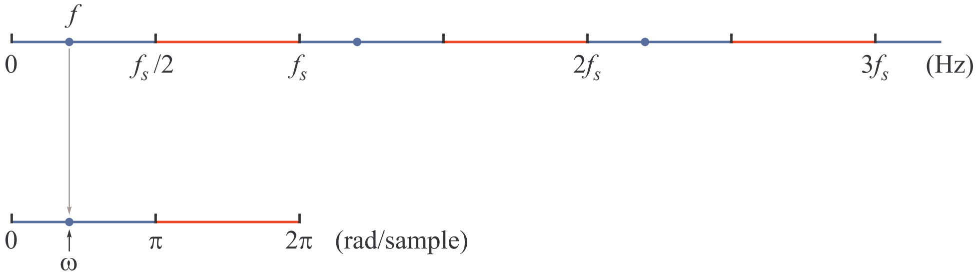

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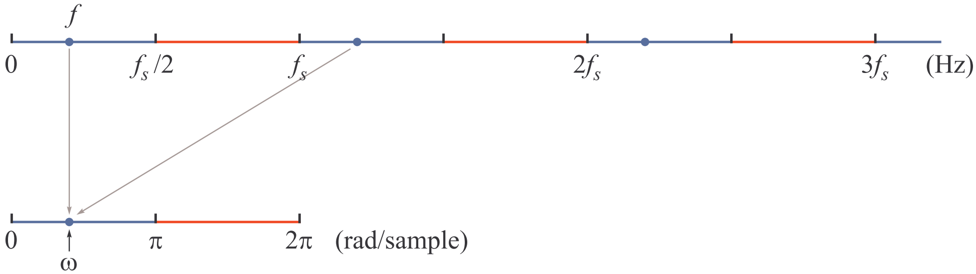

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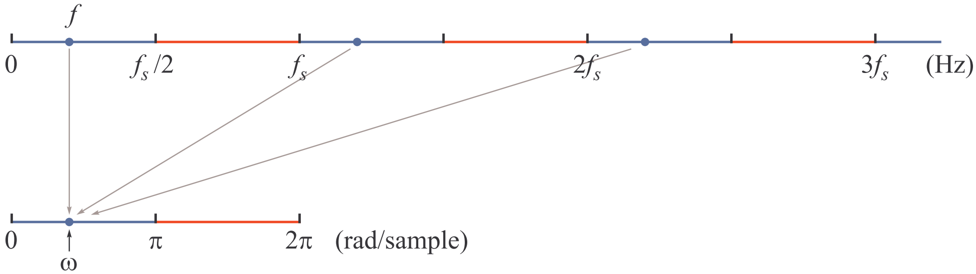

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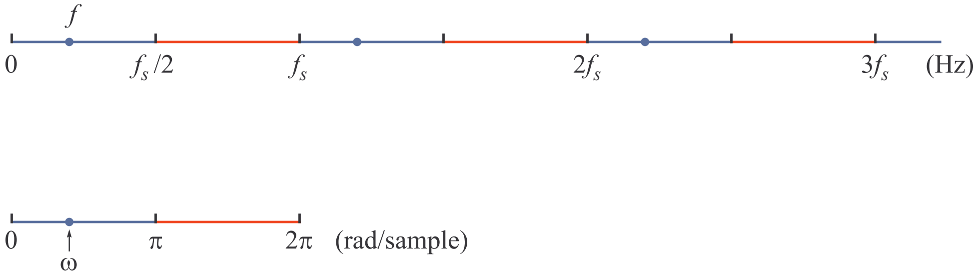

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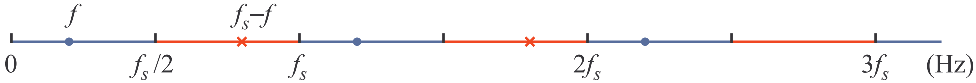


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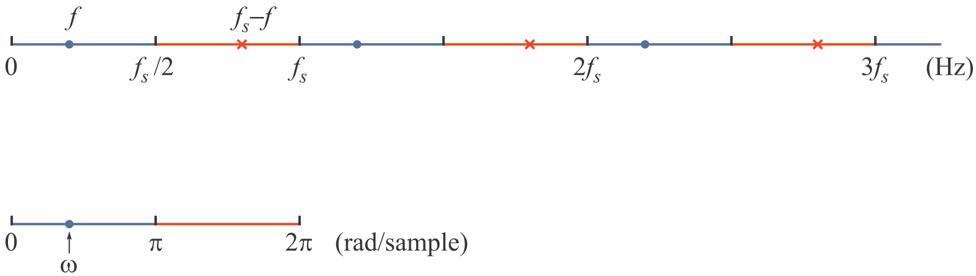

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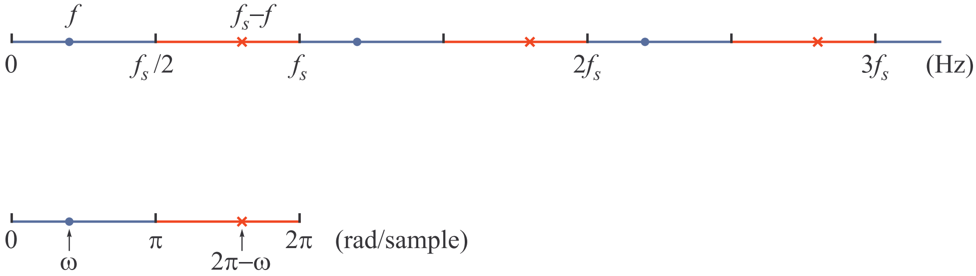

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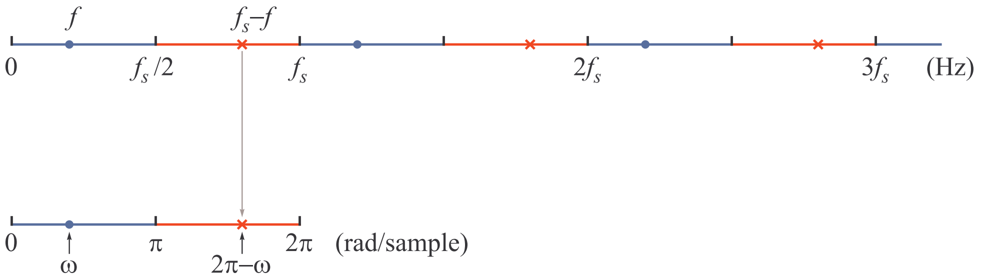

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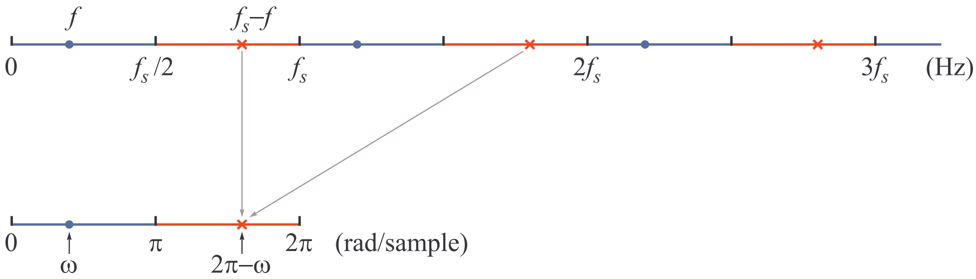

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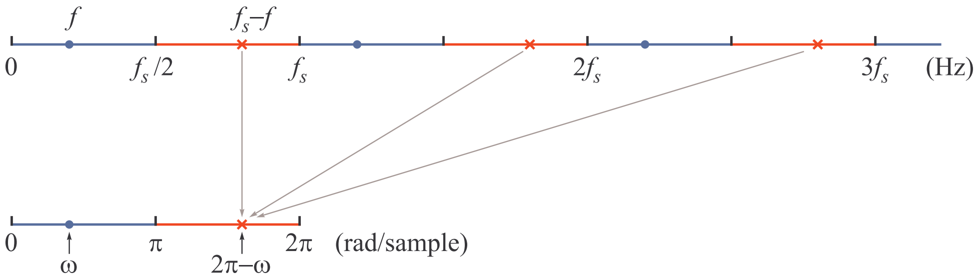

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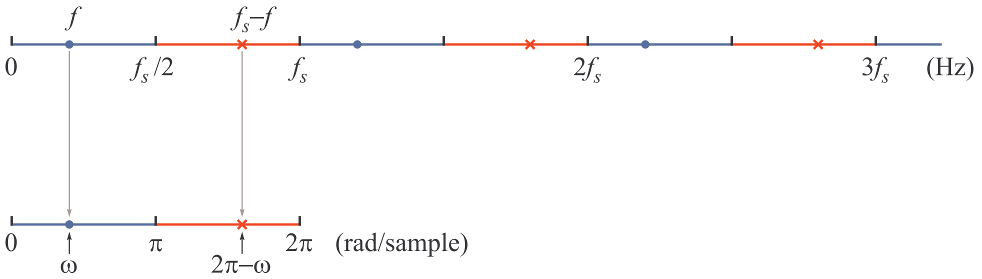

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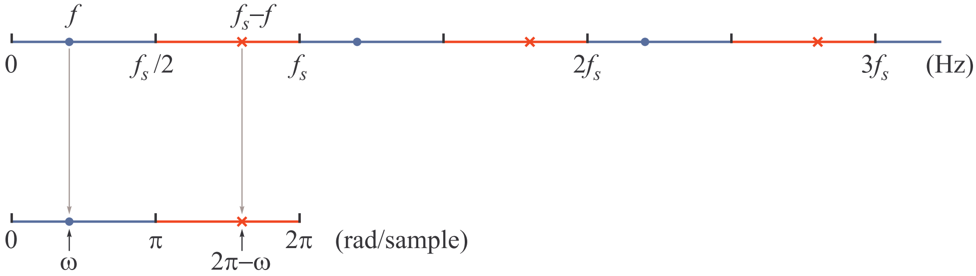


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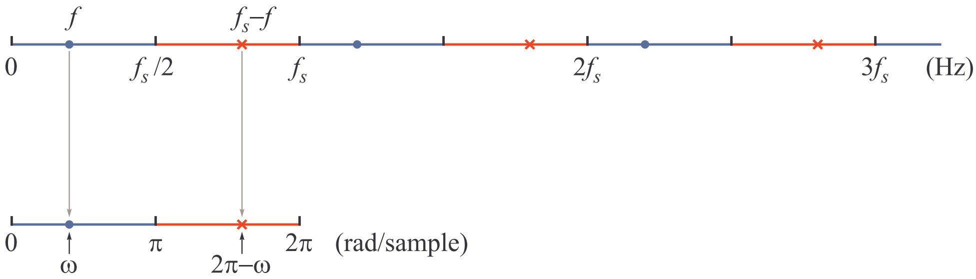
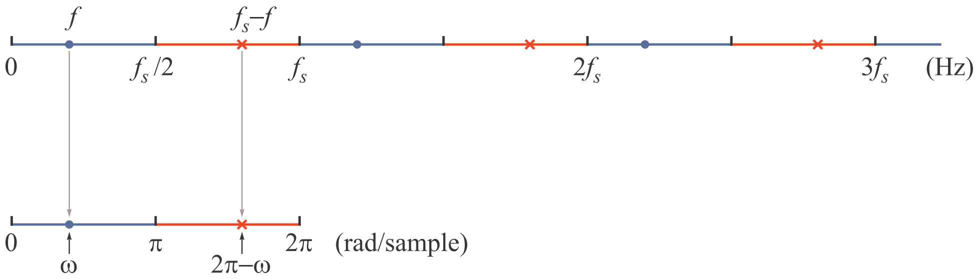



Identical sample sequences are obtained if

- $f$ and $f'$ have the same mark (● or ×)

# Aliases: Graphical Illustration

Mapping $f \to \omega$ : $\quad \omega = 2\pi(f/f_s)$

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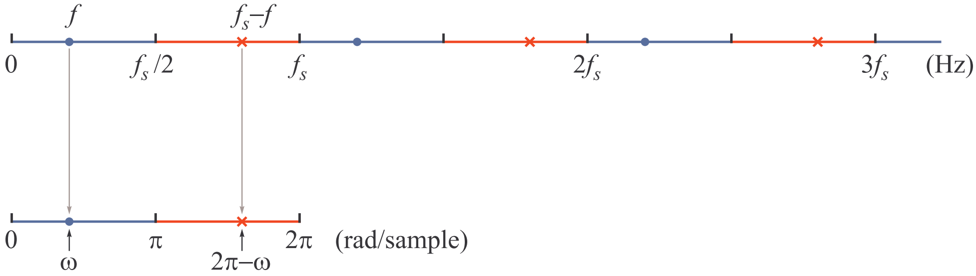


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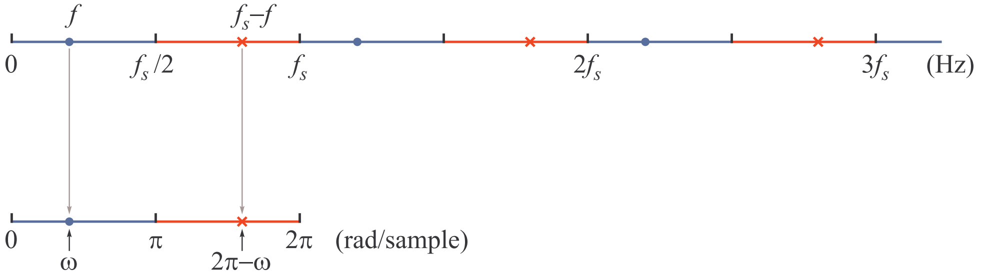


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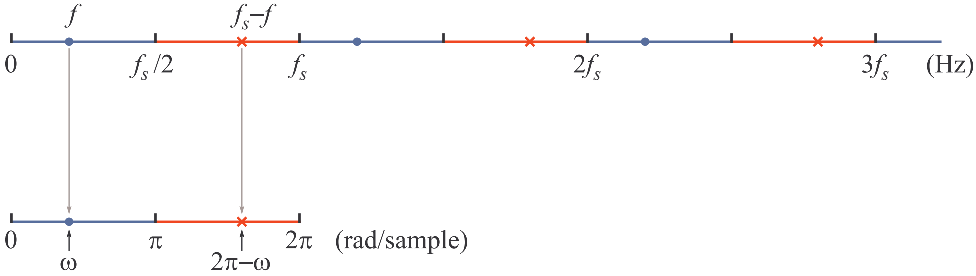


Identical sample sequences are obtained if

- $f$ and $f'$ have the same mark (● or ×) and the analog sinusoids have the same initial phase; or
- $f$ and $f'$ have different marks and the analog sinusoids have initial phases differing in sign (only).

**2.** What is the highest sampling rate $f_s$ such that $f$ and $7f$ (both positive frequencies) are aliases of each other?

A. $f_s = f$

B. $f_s = 6f$

C. $f_s = 8f$

D. $f_s = 12f$

**3.** The discrete-time sinusoid

$$x[n] \;=\; \cos(0.28\pi n - 1.7)$$

has been obtained by sampling a continuous-time sinusoid $x(t)$ at times $t = nT_s$, where $f_s = 1/T_s = 500$ samples/sec. If the frequency $f$ of $x(t)$ is known to lie in the range $[500,\ 750]$ Hz, then

A. $x(t) \;=\; \cos(1120\pi t - 1.7)$

B. $x(t) \;=\; \cos(1140\pi t - 1.7)$

C. $x(t) \;=\; \cos(1120\pi t + 1.7)$

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**4.** Once again, the discrete-time sinusoid

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A. $x(t) = \cos(1640\pi t - 1.7)$

B. $x(t) = \cos(1860\pi t - 1.7)$

C. $x(t) = \cos(1640\pi t + 1.7)$

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$\Rightarrow$ Information about the individual analog components is lost, and the analog signal cannot be reconstructed from its samples.

**5.** Which (if any) of the following continuous-time signals $x(t)$ produce

$$x(nT_s) \;=\; x[n] \;=\; 7\cos(0.2\pi n)$$

when $f_s = 1/T_s = 50$ samples/sec?

A. $x(t) \;=\; 3\cos(10\pi t) \,+\, 4\cos(190\pi t)$

B. $x(t) \;=\; \cos(10\pi t) \,+\, 6\cos(40\pi t)$

C. $x(t) \;=\; 2\cos(80\pi t) \,+\, 5\cos(120\pi t)$

D. $x(t) \;=\; 6\cos(90\pi t) \,+\, \cos(110\pi t)$

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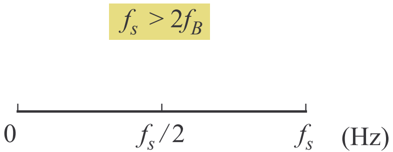

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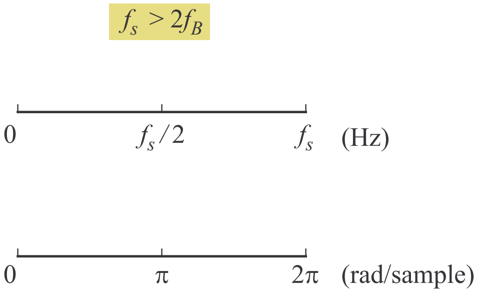

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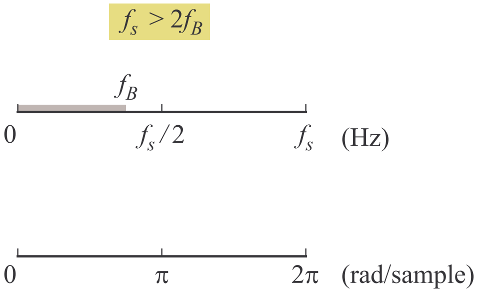

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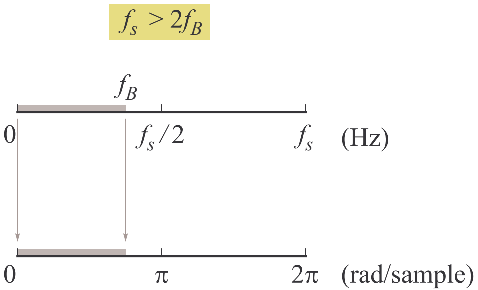

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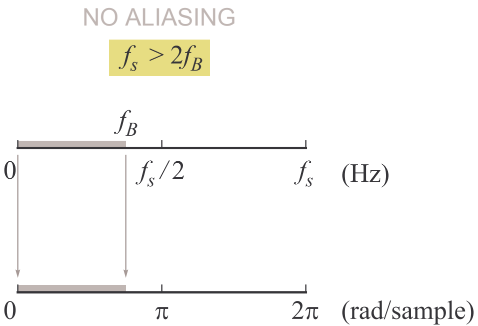

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NO ALIASING

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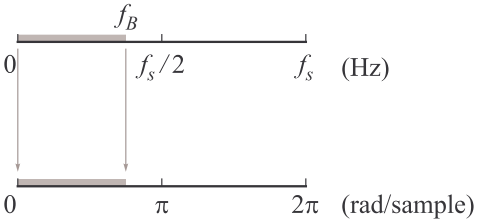


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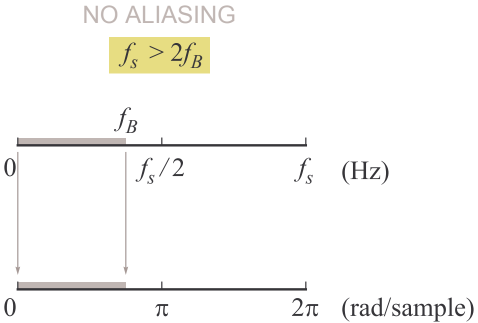


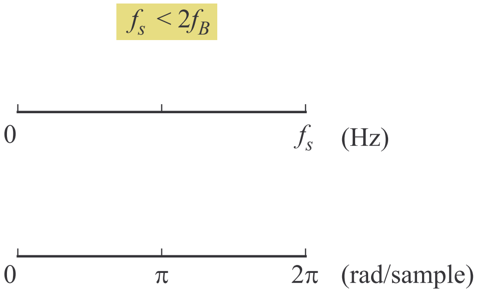

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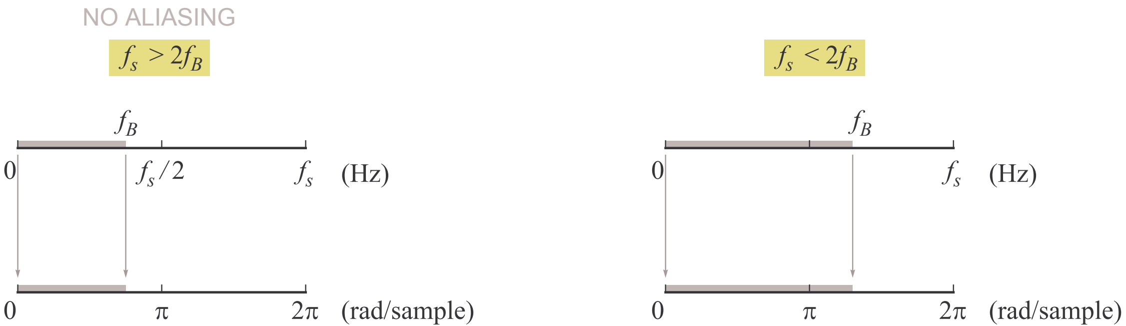

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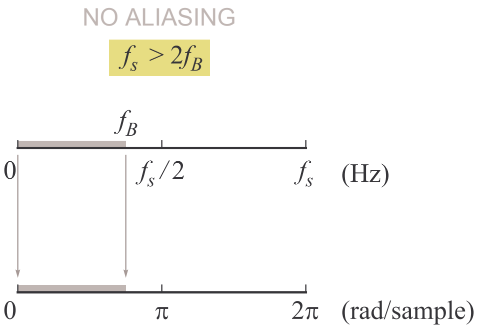


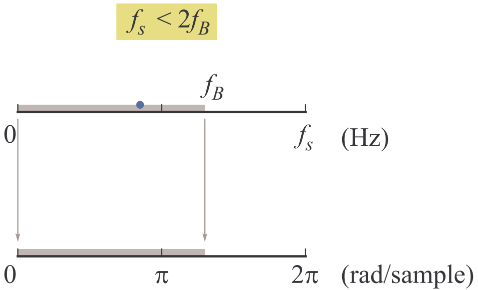

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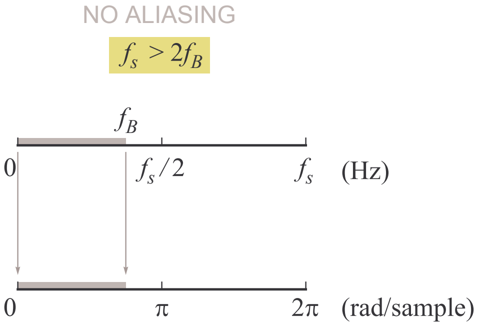


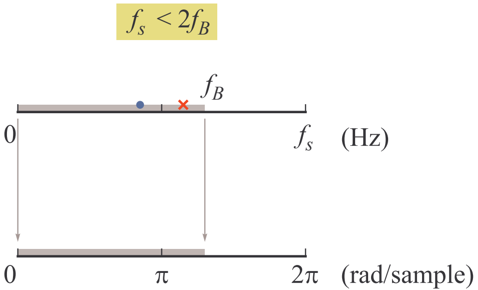

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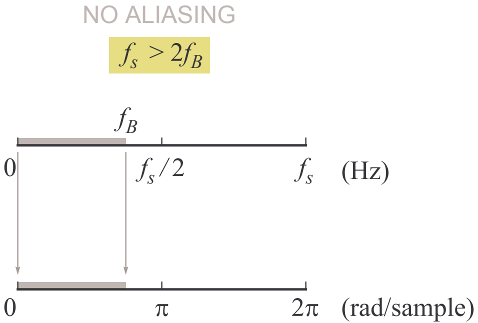


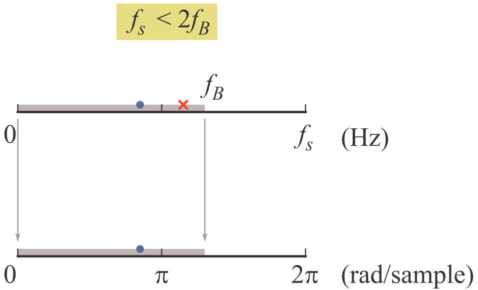

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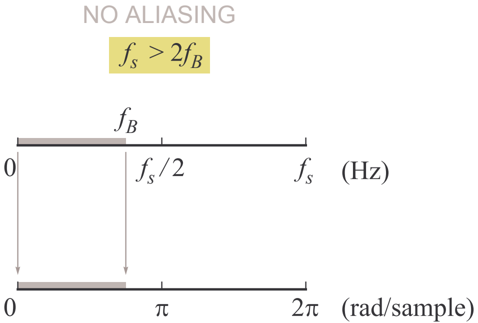


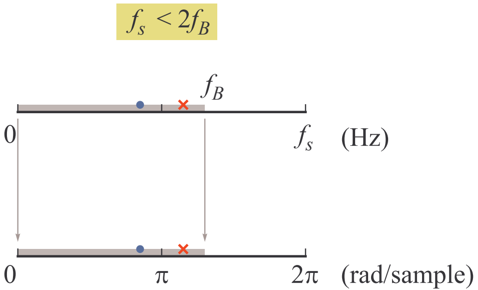

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