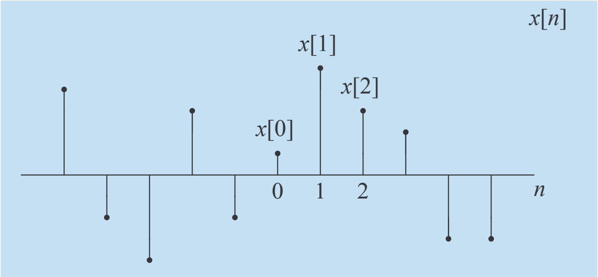
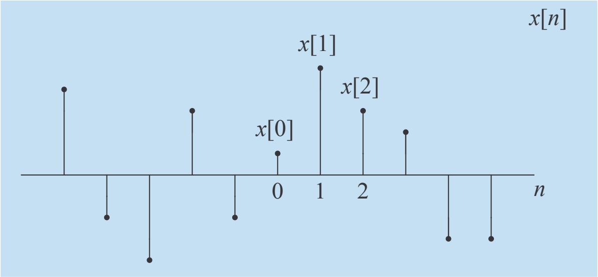
Discrete-Parameter Signals

Discrete-Parameter Signals

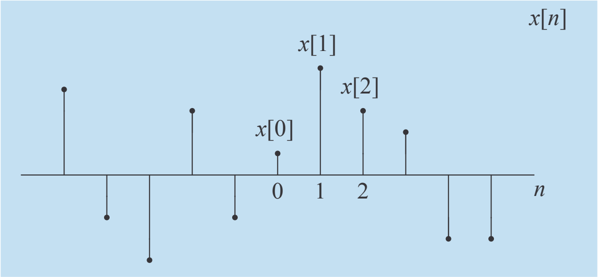


Discrete-Parameter Signals



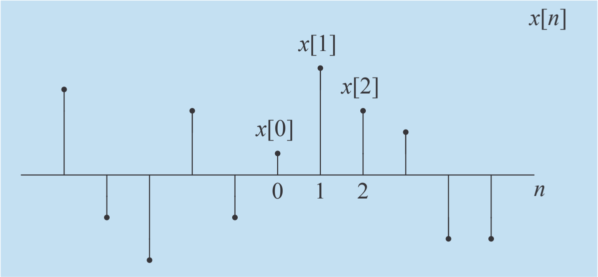
- Notation: $x[n]$ versus $x(t)$

Discrete-Parameter Signals



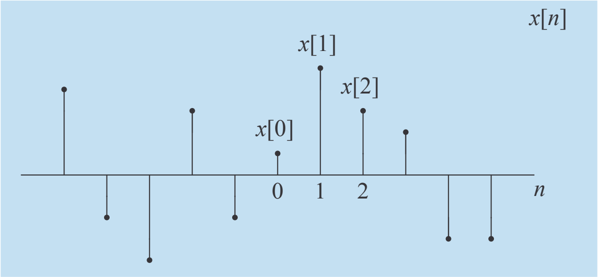
- Notation: $x[n]$ versus $x(t)$
- Typically:

Discrete-Parameter Signals



- Notation: $x[n]$ versus $x(t)$
- Typically: $x[\,\cdot\,]$ is vector

Discrete-Parameter Signals



- Notation: $x[n]$ versus $x(t)$
- Typically: $x[\,\cdot\,]$ is vector or a two-sided sequence ($n \in \mathbb{Z}$)

General Discrete-Time Sinusoid

General Discrete-Time Sinusoid

$$x[n] = A\cos(\omega n + \phi)\,, \qquad n \in \mathbb{Z}$$

General Discrete-Time Sinusoid

$$x[n] \;=\; A\cos(\omega n + \phi)\,, \qquad n \in \mathbb{Z}$$

Frequency parameter ω :

General Discrete-Time Sinusoid

$$x[n] = A\cos(\omega n + \phi), \qquad n \in \mathbb{Z}$$

Frequency parameter ω :

- is in units of radians (per sample, not second)

General Discrete-Time Sinusoid

$$x[n] \;=\; A\cos(\omega n + \phi)\,, \qquad n \in \mathbb{Z}$$

Frequency parameter ω :

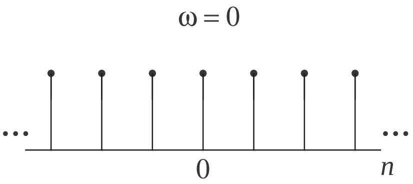
- is in units of radians (per sample, not second)
- is an angle increment

General Discrete-Time Sinusoid

$$x[n] \; = \; A\cos(\omega n + \phi)\,, \qquad n \in \mathbb{Z}$$

Frequency parameter ω :

- is in units of radians (per sample, not second)
- is an angle increment

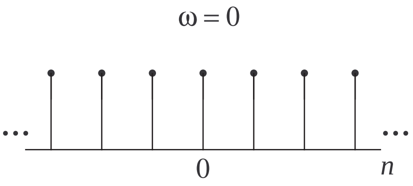


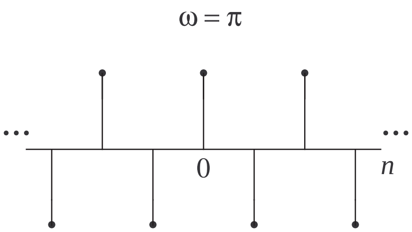
General Discrete-Time Sinusoid

$$x[n] \;=\; A\cos(\omega n + \phi)\,, \qquad n \in \mathbb{Z}$$

Frequency parameter ω :

- is in units of radians (per sample, not second)
- is an angle increment





1. How many distinct values does the discrete-time sinusoid

$$x[n] = \cos\left(\frac{\pi n}{4}\right)$$

take as n ranges over all integers (positive and negative)?

A. Four

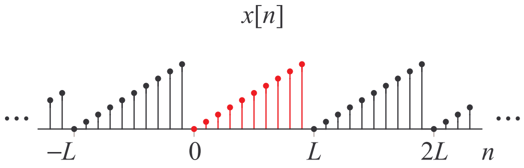
B. Five

C. Six

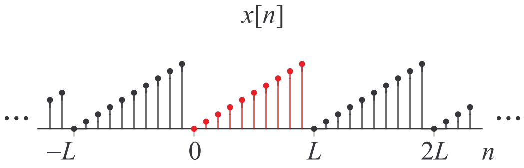
D. Eight

Periodicity

Periodicity

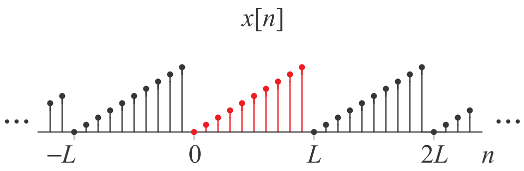


Periodicity



A sequence $x[\cdot]$ is periodic with period L samples if

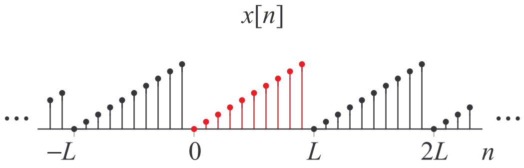
Periodicity



A sequence $x[\cdot\,]$ is periodic with period L samples if

$$x[n+L] \;=\; x[n] \qquad \text{(all } n\text{)}$$

Periodicity

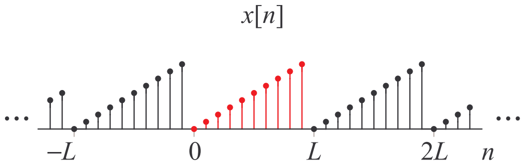


A sequence $x[\cdot]$ is periodic with period L samples if

$$x[n+L] \;=\; x[n] \qquad \text{(all } n\text{)}$$

- Fundamental period

Periodicity

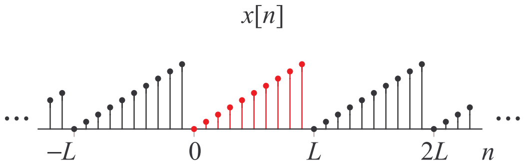


A sequence $x[\cdot]$ is periodic with period L samples if

$$x[n+L] \;=\; x[n] \qquad \text{(all } n\text{)}$$

- Fundamental period: smallest value of L for which above holds.

Periodicity

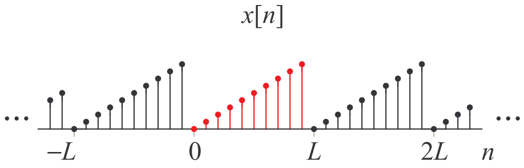


A sequence $x[\cdot]$ is periodic with period L samples if

$$x[n+L] \;=\; x[n] \qquad \text{(all } n\text{)}$$

- Fundamental period: smallest value of L for which above holds.
- Periodicity of $x[n] \;=\; \cos(\omega n + \phi)$:

Periodicity



A sequence $x[\cdot]$ is periodic with period L samples if

$$x[n+L] \;=\; x[n] \qquad \text{(all } n\text{)}$$

- Fundamental period: smallest value of L for which above holds.
- Periodicity of $x[n] = \cos(\omega n + \phi)$: depends on the value of ω.

Periodicity of $\cos(\omega n + \phi)$

Periodicity of $\cos(\omega n + \phi)$

angle covered in L time instants $=$ whole number of cycles

Periodicity of $\cos(\omega n + \phi)$

$$\text{angle covered in } L \text{ time instants} \quad = \quad \text{whole number of cycles}$$

$$\omega L \quad = \quad k \cdot 2\pi$$

Periodicity of $\cos(\omega n + \phi)$

$$
\begin{aligned}
\text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\
\omega L &= k \cdot 2\pi \\
\omega &= \frac{k}{L} \cdot 2\pi
\end{aligned}
$$

Periodicity of $\cos(\omega n + \phi)$

$$\begin{aligned} \text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\ \omega L &= k \cdot 2\pi \\ \omega &= \frac{k}{L} \cdot 2\pi \end{aligned}$$

Reduced fraction $k/L \;\Rightarrow\;$ fundamental period $=\; L$

Periodicity of $\cos(\omega n + \phi)$

$$\begin{aligned}\text{angle covered in } L \text{ time instants} &= \text{whole number of cycles}\\ \omega L &= k \cdot 2\pi \\ \omega &= \frac{k}{L} \cdot 2\pi\end{aligned}$$

Reduced fraction $k/L \;\Rightarrow\;$ fundamental period $= L$

Example: $\omega = \dfrac{3\pi}{4}$

Periodicity of $\cos(\omega n + \phi)$

$$\begin{aligned} \text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\ \omega L &= k \cdot 2\pi \\ \omega &= \frac{k}{L} \cdot 2\pi \end{aligned}$$

Reduced fraction $k/L \;\Rightarrow$ fundamental period $= L$

Example: $\omega = \dfrac{3\pi}{4} = \dfrac{3}{8} \cdot 2\pi$

Periodicity of $\cos(\omega n + \phi)$

$$\begin{aligned} \text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\ \omega L &= k \cdot 2\pi \\ \omega &= \frac{k}{L} \cdot 2\pi \end{aligned}$$

Reduced fraction $k/L \;\Rightarrow\;$ fundamental period $= L$

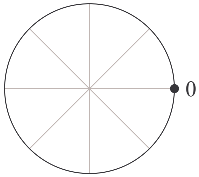
Example: $\omega = \frac{3\pi}{4} = \frac{3}{8} \cdot 2\pi \quad \Rightarrow \quad L = 8$ samples

Periodicity of $\cos(\omega n + \phi)$

$$
\begin{aligned}
\text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\
\omega L &= k \cdot 2\pi \\
\omega &= \frac{k}{L} \cdot 2\pi
\end{aligned}
$$

Reduced fraction $k/L \;\Rightarrow$ fundamental period $= L$

Example: $\omega = \frac{3\pi}{4} = \frac{3}{8} \cdot 2\pi \quad \Rightarrow \quad L = 8$ samples

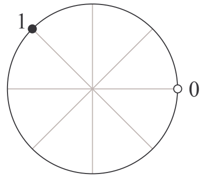


Periodicity of $\cos(\omega n + \phi)$

$$\begin{aligned} \text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\ \omega L &= k \cdot 2\pi \\ \omega &= \frac{k}{L} \cdot 2\pi \end{aligned}$$

Reduced fraction $k/L \;\Rightarrow$ fundamental period $= L$

Example: $\omega = \dfrac{3\pi}{4} = \dfrac{3}{8} \cdot 2\pi \quad \Rightarrow \quad L = 8$ samples

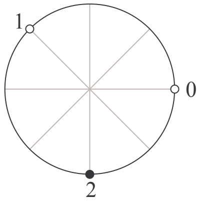


Periodicity of $\cos(\omega n + \phi)$

$$\begin{aligned}
\text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\
\omega L &= k \cdot 2\pi \\
\omega &= \frac{k}{L} \cdot 2\pi
\end{aligned}$$

Reduced fraction $k/L \Rightarrow$ fundamental period $= L$

Example: $\omega = \dfrac{3\pi}{4} = \dfrac{3}{8} \cdot 2\pi \Rightarrow L = 8$ samples



Periodicity of $\cos(\omega n + \phi)$

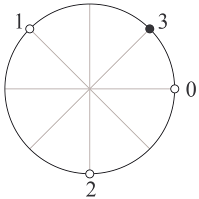
$$\text{angle covered in } L \text{ time instants} \;=\; \text{whole number of cycles}$$

$$\omega L \;=\; k \cdot 2\pi$$

$$\omega \;=\; \frac{k}{L} \cdot 2\pi$$

Reduced fraction $k/L \;\Rightarrow$ fundamental period $= L$

Example: $\omega = \frac{3\pi}{4} = \frac{3}{8} \cdot 2\pi \;\Rightarrow\; L = 8$ samples

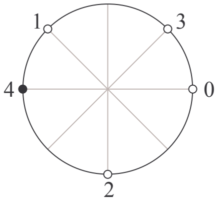


Periodicity of $\cos(\omega n + \phi)$

$$\begin{aligned} \text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\ \omega L &= k \cdot 2\pi \\ \omega &= \frac{k}{L} \cdot 2\pi \end{aligned}$$

Reduced fraction $k/L \;\Rightarrow$ fundamental period $= L$

Example: $\omega = \frac{3\pi}{4} = \frac{3}{8} \cdot 2\pi \;\Rightarrow\; L = 8$ samples

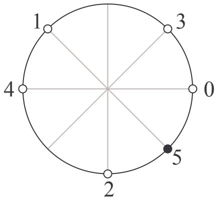


Periodicity of $\cos(\omega n + \phi)$

$$
\begin{aligned}
\text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\
\omega L &= k \cdot 2\pi \\
\omega &= \frac{k}{L} \cdot 2\pi
\end{aligned}
$$

Reduced fraction $k/L \;\Rightarrow$ fundamental period $= L$

Example: $\omega = \dfrac{3\pi}{4} = \dfrac{3}{8} \cdot 2\pi \quad \Rightarrow \quad L = 8$ samples

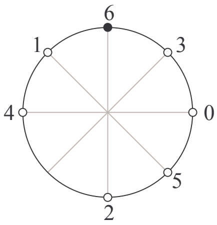


Periodicity of $\cos(\omega n + \phi)$

$$\begin{aligned} \text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\ \omega L &= k \cdot 2\pi \\ \omega &= \frac{k}{L} \cdot 2\pi \end{aligned}$$

Reduced fraction $k/L \;\Rightarrow$ fundamental period $= L$

Example: $\omega = \dfrac{3\pi}{4} = \dfrac{3}{8} \cdot 2\pi \;\Rightarrow\; L = 8$ samples



Periodicity of $\cos(\omega n + \phi)$

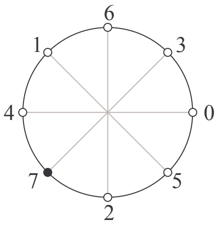
$$\text{angle covered in } L \text{ time instants} \quad = \quad \text{whole number of cycles}$$

$$\omega L = k \cdot 2\pi$$

$$\omega = \frac{k}{L} \cdot 2\pi$$

Reduced fraction $k/L \Rightarrow$ fundamental period $= L$

Example: $\omega = \dfrac{3\pi}{4} = \dfrac{3}{8} \cdot 2\pi \quad \Rightarrow \quad L = 8$ samples

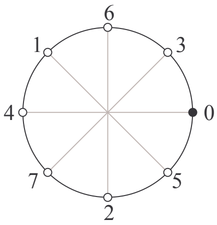


Periodicity of $\cos(\omega n + \phi)$

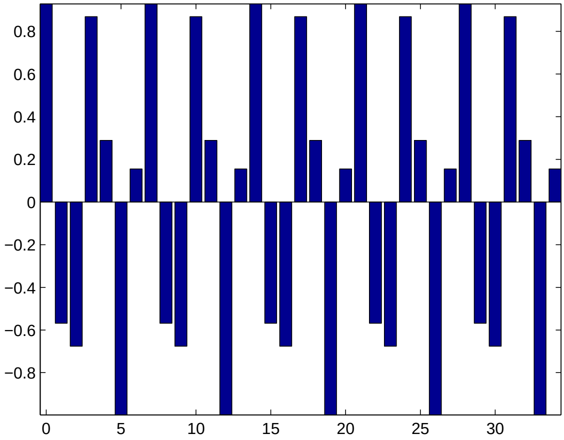
$$\begin{aligned} \text{angle covered in } L \text{ time instants} &= \text{whole number of cycles} \\ \omega L &= k \cdot 2\pi \\ \omega &= \frac{k}{L} \cdot 2\pi \end{aligned}$$

Reduced fraction $k/L \;\Rightarrow$ fundamental period $= \; L$

Example: $\omega = \dfrac{3\pi}{4} = \dfrac{3}{8} \cdot 2\pi \;\Rightarrow\; L = 8$ samples



2. Shown below is a bar plot of the discrete-time sinusoid $\cos(\omega n + \phi)$. Which of the following values of ω is most consistent with this plot?



A. $4\pi/7$ B. $5\pi/7$ C. $4\pi/9$ D. $5\pi/9$

Effective Range of ω

Effective Range of ω

- Unlike Ω (rad/sec),

Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.

Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid),

Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

Effective Range of ω

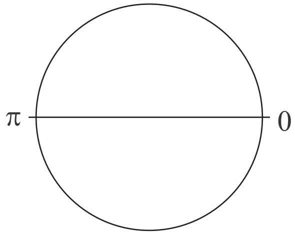
- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

$$\begin{aligned}\cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi)\end{aligned}$$

Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

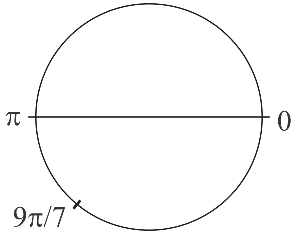
$$\begin{aligned}\cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi)\end{aligned}$$



Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

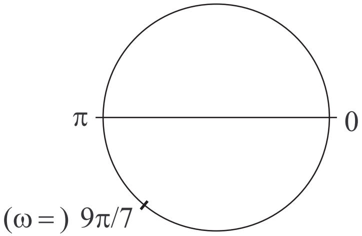
$$\begin{aligned} \cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi) \end{aligned}$$



Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

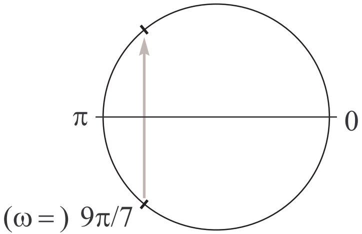
$$\begin{aligned} \cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi) \end{aligned}$$



Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

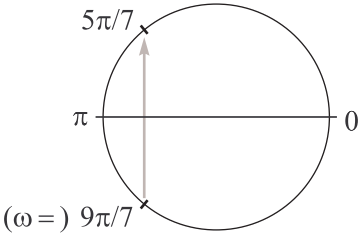
$$\begin{aligned}\cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi)\end{aligned}$$



Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

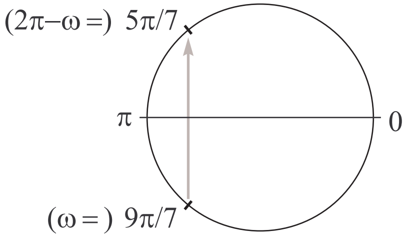
$$\begin{aligned}\cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi)\end{aligned}$$



Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

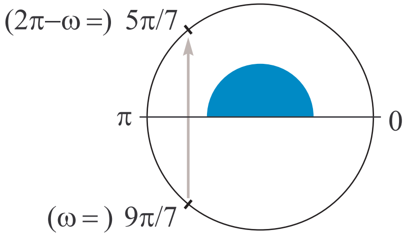
$$\begin{aligned}\cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi)\end{aligned}$$



Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

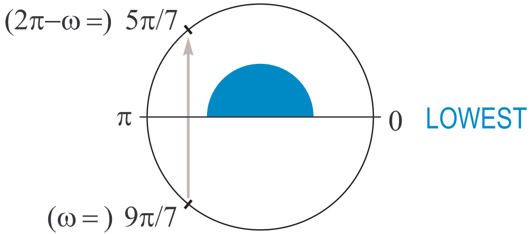
$$\begin{aligned} \cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi) \end{aligned}$$



Effective Range of ω

- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

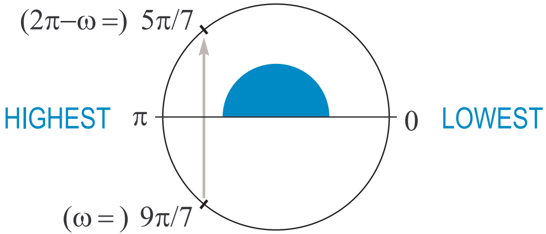
$$\begin{aligned}\cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi)\end{aligned}$$



Effective Range of ω

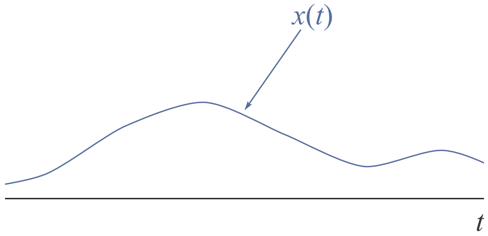
- Unlike Ω (rad/sec), ω (rad) is limited to $[0, 2\pi)$.
- For $x[n] = \cos(\omega n + \phi)$ (real sinusoid), ω is further limited to $[0, \pi]$:

$$\begin{aligned}\cos(\omega n + \phi) &= \cos(-\omega n - \phi) \\ &= \cos((2\pi - \omega)n - \phi)\end{aligned}$$

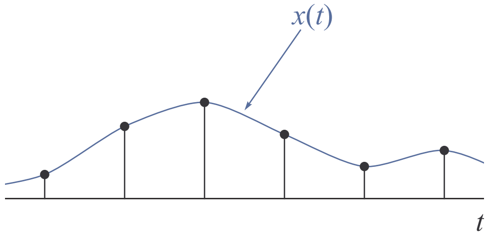


Sampling of Continuous-Time Signals

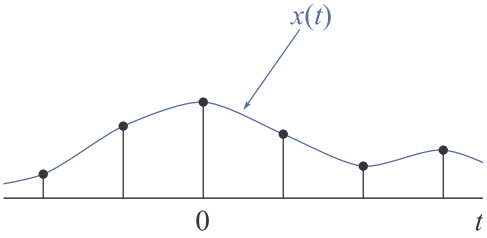
Sampling of Continuous-Time Signals



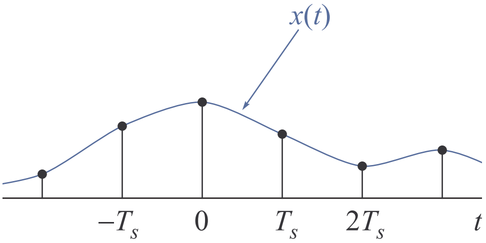
Sampling of Continuous-Time Signals



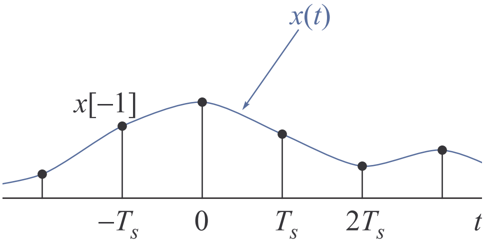
Sampling of Continuous-Time Signals



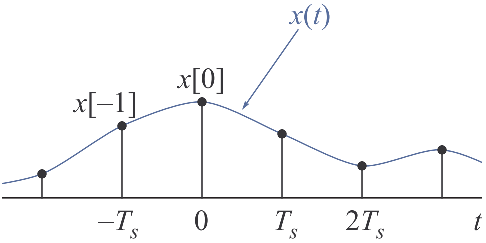
Sampling of Continuous-Time Signals



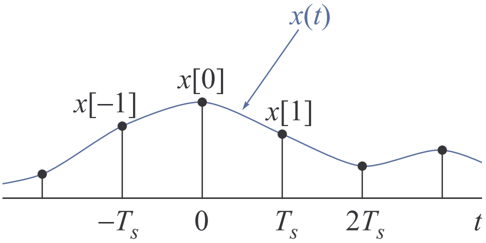
Sampling of Continuous-Time Signals



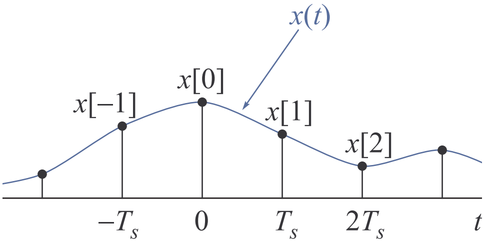
Sampling of Continuous-Time Signals



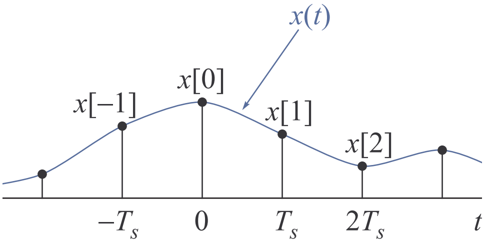
Sampling of Continuous-Time Signals



Sampling of Continuous-Time Signals

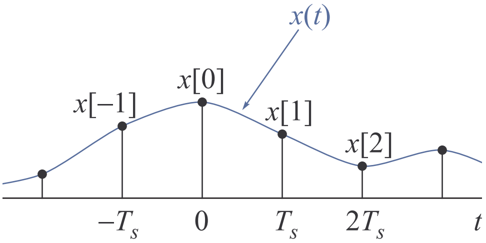


Sampling of Continuous-Time Signals



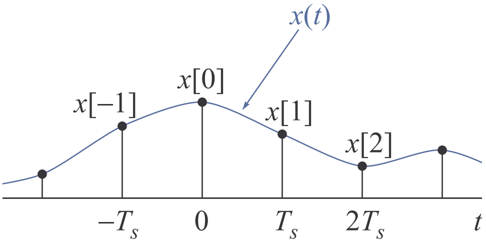
- T_s = sampling period (sec)

Sampling of Continuous-Time Signals



- $T_s \;=\;$ sampling period (sec)
- $f_s \;=\; 1/T_s \;=\;$ sampling rate (samples/sec)

Sampling of Continuous-Time Signals



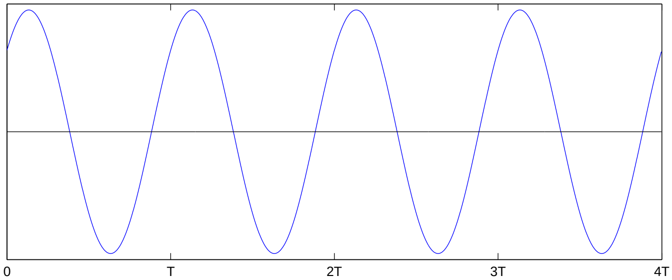
- $T_s \;=\;$ sampling period (sec)
- $f_s \;=\; 1/T_s \;=\;$ sampling rate (samples/sec)

Sample sequence $x[n]$ is given by

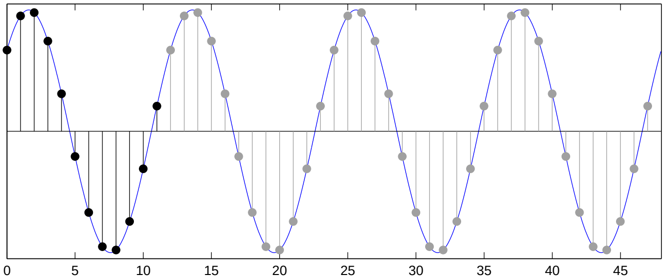
$$x[n] \;=\; x(nT_s) \qquad \text{(all } n\text{)}$$

Sampling $x(t) = A\cos(\Omega t + \phi)$

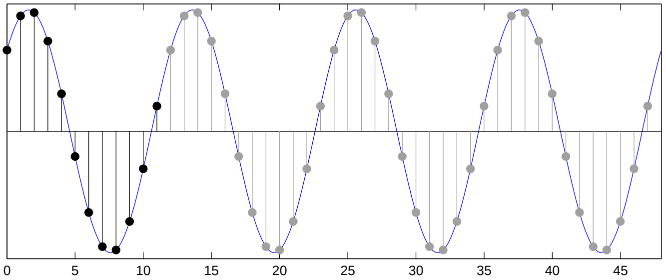
Sampling $x(t) = A\cos(\Omega t + \phi)$



Sampling $x(t) = A\cos(\Omega t + \phi)$

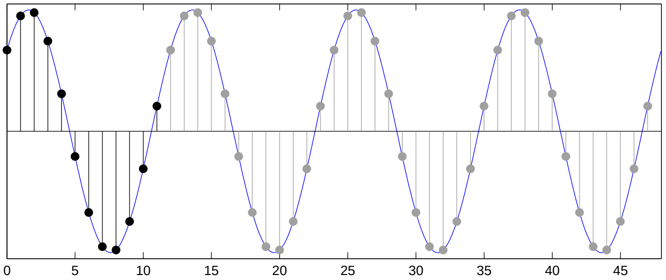


Sampling $x(t) = A\cos(\Omega t + \phi)$



$$x[n] = x(nT_s) = A\cos(\omega n + \phi)$$

Sampling $x(t) = A\cos(\Omega t + \phi)$

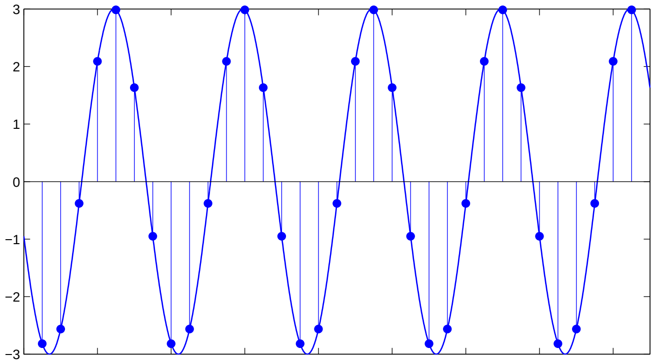


$$x[n] = x(nT_s) = A\cos(\omega n + \phi)$$

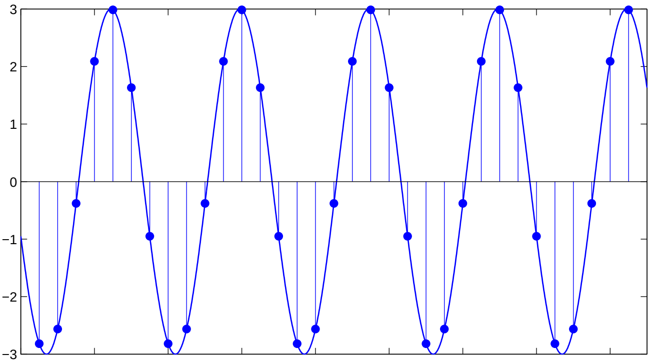
where

$$\omega = \Omega T_s = 2\pi \cdot \frac{T_s}{T}$$

3 and 4. The continuous-time sinusoid $x(t) = 3\cos(\Omega t + \phi)$ is plotted below (in black). The stem plot (in blue) is the sequence of samples $x[n] = x(nT_s)$.



3 and 4. The continuous-time sinusoid $x(t) = 3\cos(\Omega t + \phi)$ is plotted below (in black). The stem plot (in blue) is the sequence of samples $x[n] = x(nT_s)$.



Write an equation for $x[n]$ that contains neither Ω nor T_s.

5. Let

$$x(t) = \cos(200\pi t + 1.7) ,$$

where t is in seconds. For which (one or more) of the following values of T_s is the sample sequence $x[n] = x(nT_s)$ given by the equation

$$x[n] = \cos(0.7\pi n - 1.7) ?$$

A. 3.5 ms

B. 6.5 ms

C. 13.5 ms

D. 16.5 ms