The Complex Exponential

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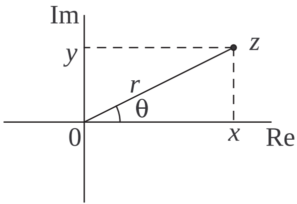

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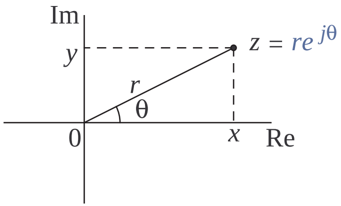

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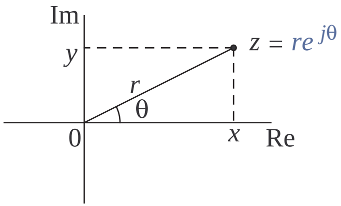

$$e^{j\theta} = \cos\theta + j\sin\theta$$

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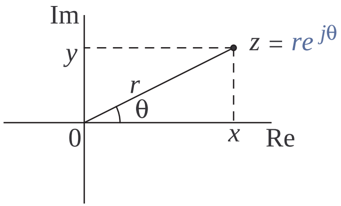

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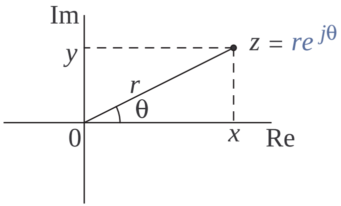

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(see L3.2 2 3)

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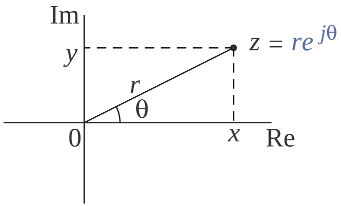

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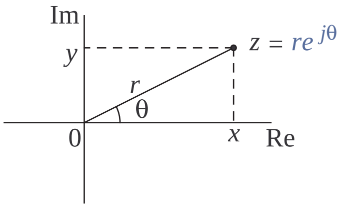

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$$e^t, \;\; \cos\theta \;\; \text{and} \;\; \sin\theta$$

(see L3.2 2 3)

- Consistent with multiplication/division formulas in polar coordinates, e.g.,

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) \; = \; r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

1. The complex number

$$z = e^{j\pi/3} + e^{-j\pi/3} + j(e^{j\pi/6} - e^{-j\pi/6})$$

also equals

A. 0

B. 2

C. $1 + j$

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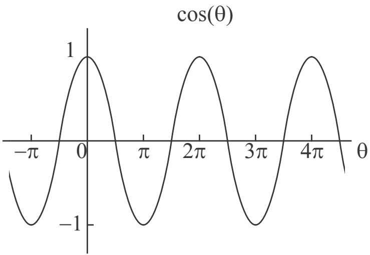
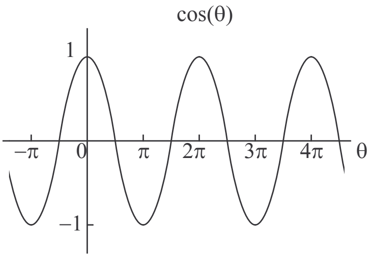

cos(θ)
1
−1
−π
0
π
2π
3π
4π
θ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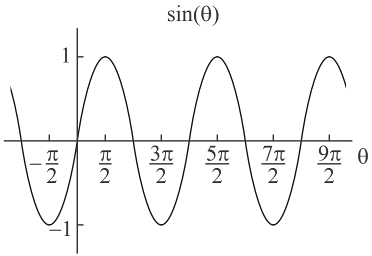
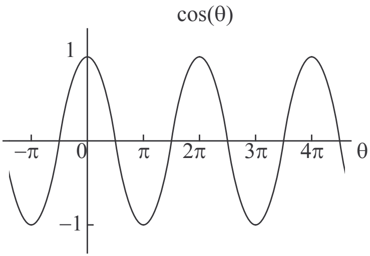
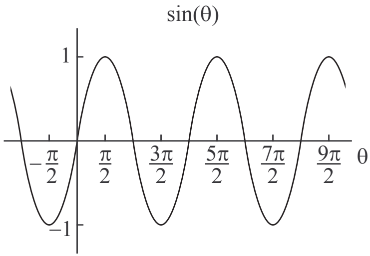
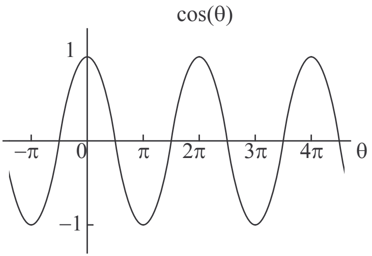
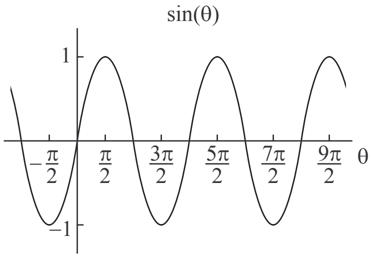

sin(θ)
1
−1
− π/2
π/2
3π/2
5π/2
7π/2
9π/2
θ





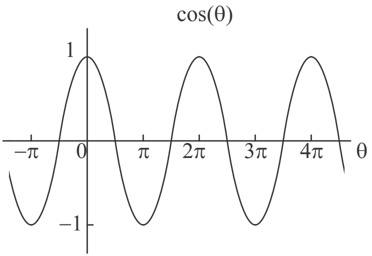
Periodicity

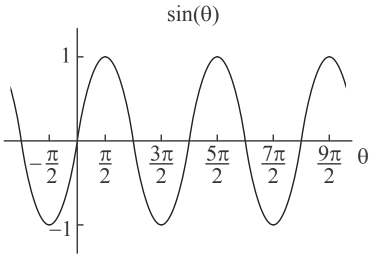




Periodicity:

$$\cos(\theta + 2\pi) \;=\; \cos\theta$$





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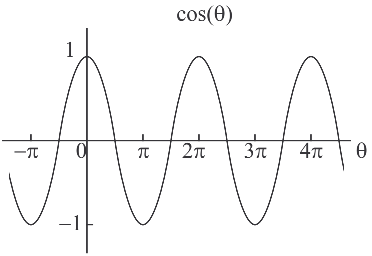

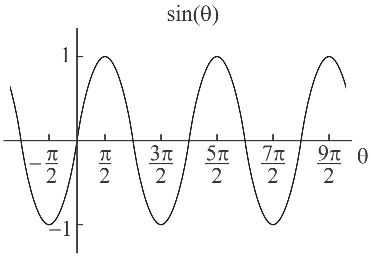

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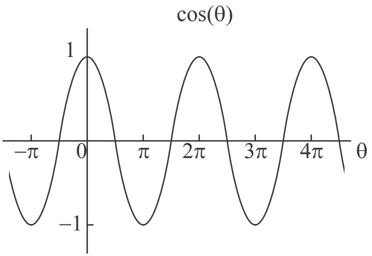
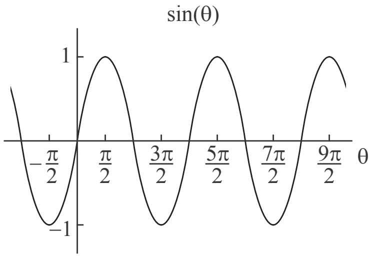
Even/Odd symmetry





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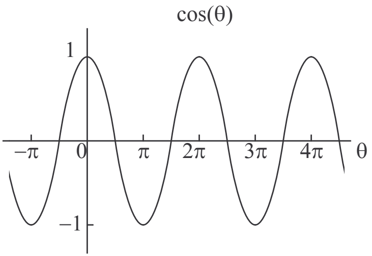

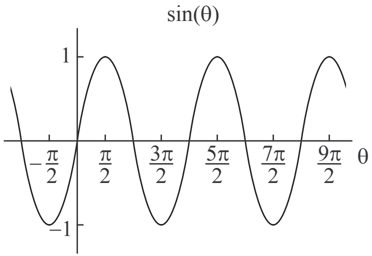

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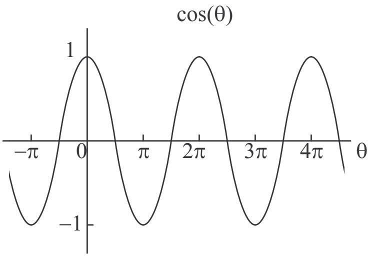

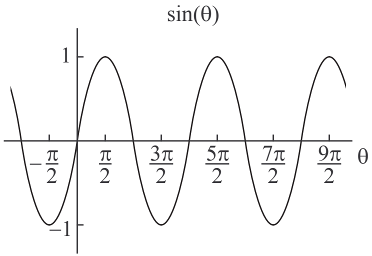

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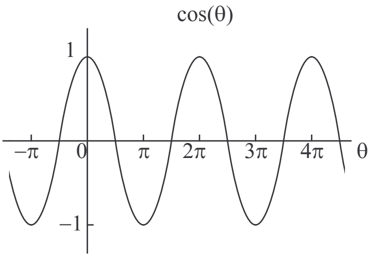

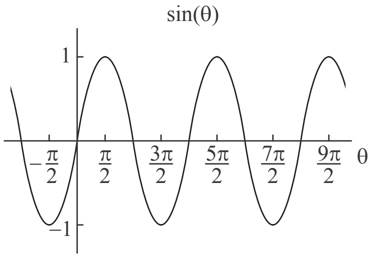

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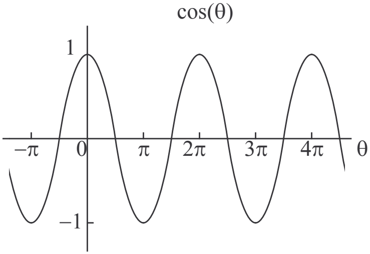

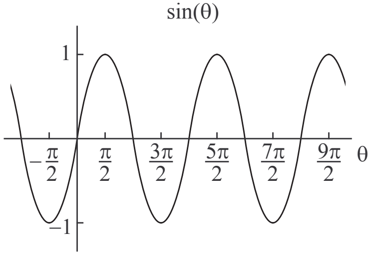

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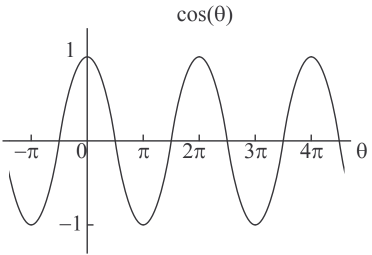

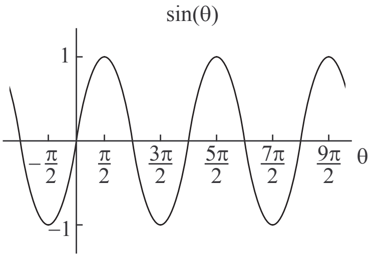

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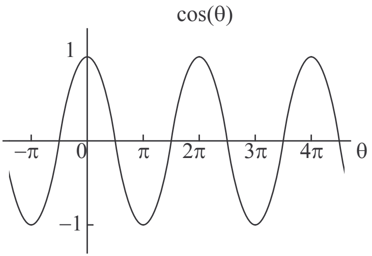
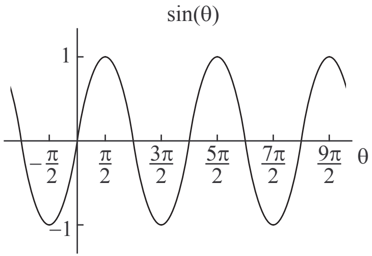
Half-cycle shifts





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Half-cycle shifts:

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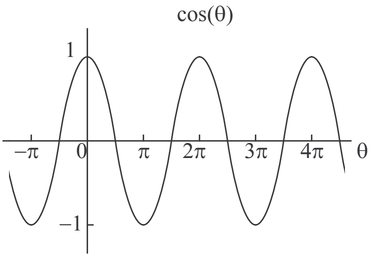

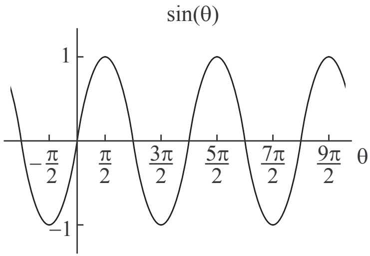

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General Continuous-Time Sinusoid

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$$\Omega \;=\; 2\pi f \;=\; \frac{2\pi}{T}$$

General Continuous-Time Sinusoid

$$x(t) = A\cos(\Omega t + \phi)$$

Parameters:

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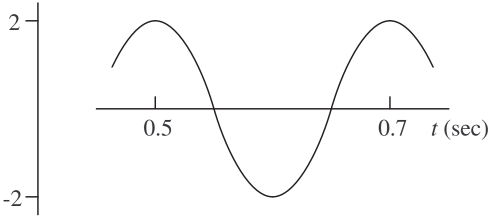
- Angular frequency Ω (rad/sec)

  Also: cyclic frequency f (Hz); period T (sec)

  $$\Omega = 2\pi f = \frac{2\pi}{T}$$

- Initial phase ϕ (rad)

2. Shown below is a partial plot of the sinusoid $x(t) = 2\cos(\Omega t + \phi)$.



Which of the following values of ϕ is correct?

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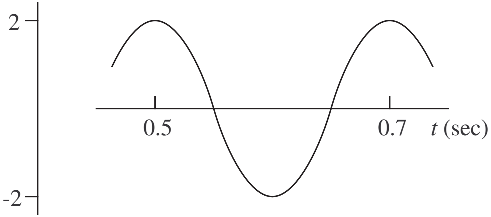

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See also graphing example in L3.3 4 .

3. $x(t) = A\cos(\Omega t + \phi)$ is such that

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Based on the given information, determine A, Ω and ϕ.

Addition of Sinusoids Using Phasors

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$$x(t) \;=\; A\cos(\Omega t + \phi)$$

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$$x(t) \;=\; A\cos(\Omega t + \phi) \;=\; \Re e\left\{Ae^{j(\Omega t+\phi)}\right\} \;=\; \Re e\{z(t)\}$$

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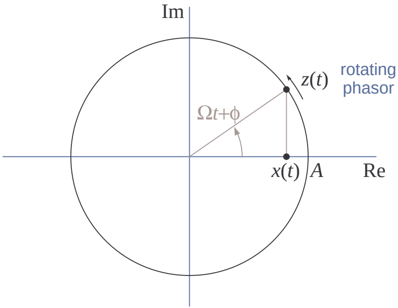

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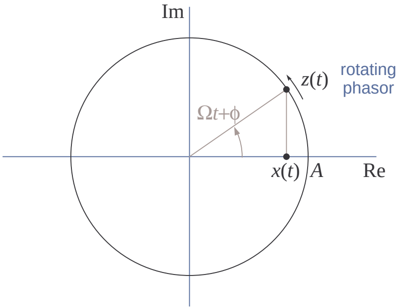

$$A_1\cos(\Omega t + \phi_1) + A_2\cos(\Omega t + \phi_2) =$$

Addition of Sinusoids Using Phasors

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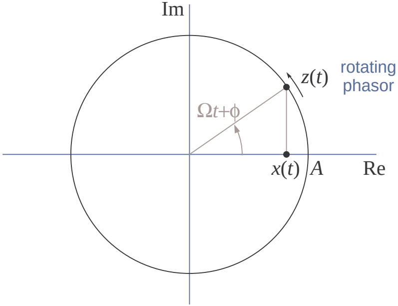

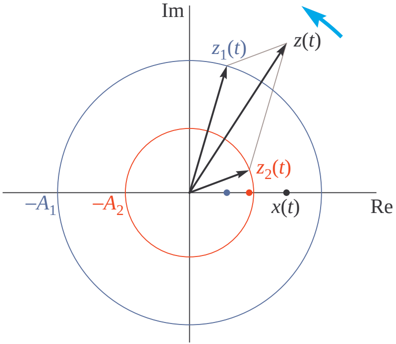

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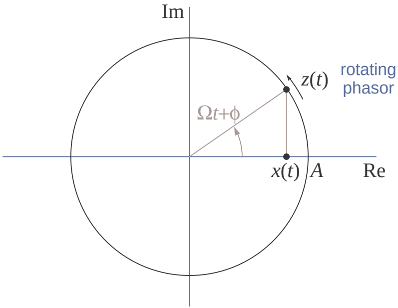

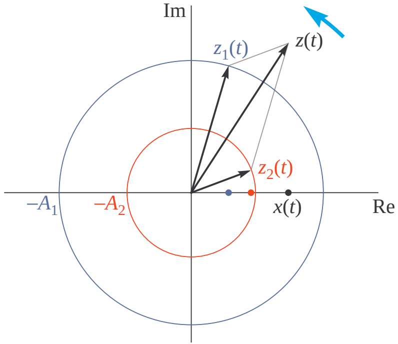

$$A_1\cos(\Omega t + \phi_1) \;+\; A_2\cos(\Omega t + \phi_2) \;=\; A\cos(\Omega t + \phi)$$

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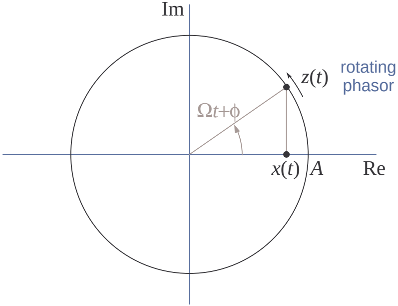
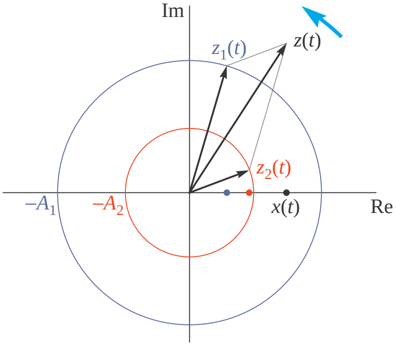
$$x(t) = A\cos(\Omega t + \phi) = \Re e\left\{Ae^{j(\Omega t+\phi)}\right\} = \Re e\{z(t)\}$$





$$A_1\cos(\Omega t + \phi_1) + A_2\cos(\Omega t + \phi_2) = A\cos(\Omega t + \phi),$$

where

$$Ae^{j\phi} = A_1 e^{j\phi_1} + A_2 e^{j\phi_2}$$

4. The sum

$$A\cos\left(\Omega t + \frac{\pi}{6}\right) + A\cos\left(\Omega t - \frac{\pi}{6}\right)$$

also equals

A. $2A\cos(\Omega t)$

B. $A\cos(\Omega t)$

C. $\sqrt{3}A\cos(\Omega t)$

D. $A\cos(2\Omega t)$

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$$A\cos\left(\Omega t + \frac{\pi}{6}\right) + A\cos\left(\Omega t - \frac{\pi}{6}\right)$$

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See L4.1 3 for a more numerically involved example.