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(Vector of z is rotated by angle of w.)

3. If

- z_1 has modulus 1 and angle $-\pi/3$; and
- z_2 has modulus 2 and angle $\pi/12$,

then $z_1 z_2$ equals

A. 2

B. $-2j$

C. $\sqrt{2} + j\sqrt{2}$

D. $\sqrt{2} - j\sqrt{2}$

4. If $\angle z = 3\pi/7$, for which (one or more) of the following choices of n is z^n real-valued?

A. $n = 28$

B. $n = 17$

C. $n = 3$

D. $n = 35$

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Distributivity

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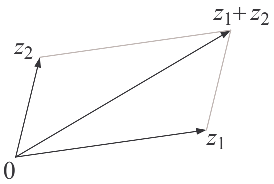
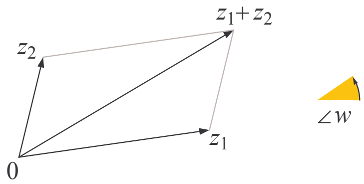


Distributivity

Assume $|w| = 1$.



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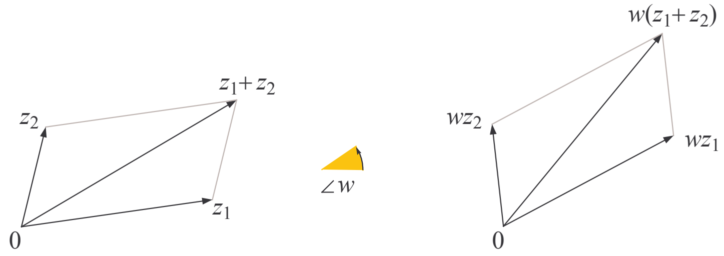

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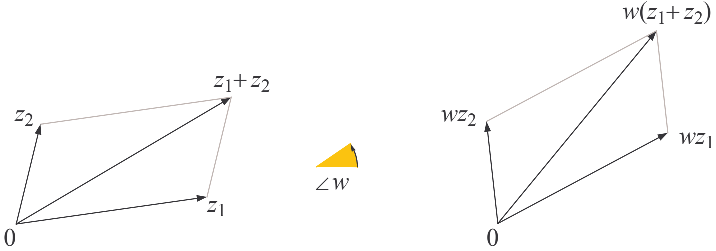

$$w(z_1 + z_2) = wz_1 + wz_2$$

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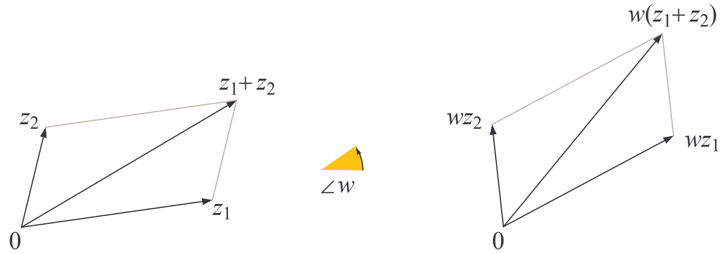

$$w(z_1 + z_2) = wz_1 + wz_2$$

$$aw(z_1 + z_2) = awz_1 + awz_2 \qquad (a \geq 0)$$

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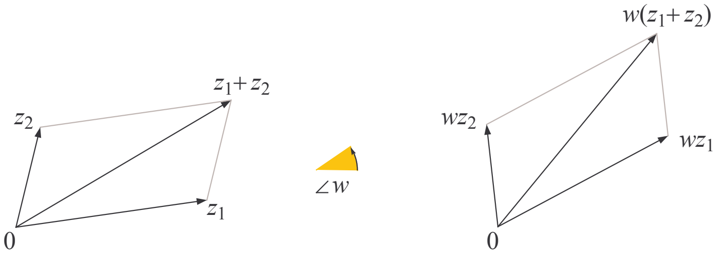

$$w(z_1 + z_2) = wz_1 + wz_2$$

$$aw(z_1 + z_2) = awz_1 + awz_2 \qquad (a \geq 0)$$

$$z_0(z_1 + z_2) = z_0 z_1 + z_0 z_2$$

Complex Product in Cartesian Coordinates

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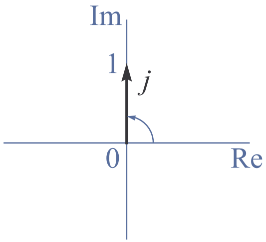

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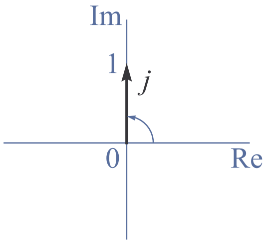

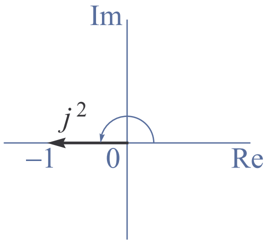

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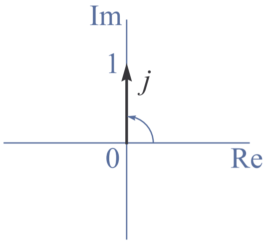

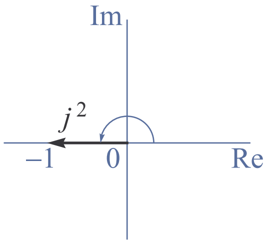

$$j^2 = -1$$

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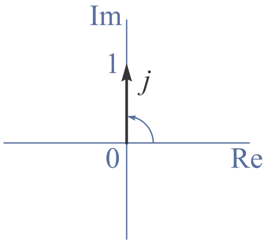

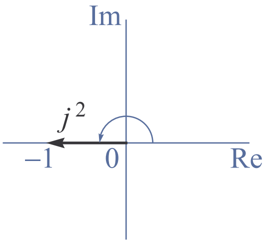

$$j^2 = -1$$

Using distributivity,

$$(a + jb)(c + jd) =$$

1. The product $(2 - j)(1 - 3j)$ equals

A. $-1 - 5j$

B. $-5 - 5j$

C. $-1 - 7j$

D. $5 - 7j$

Complex Division

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$$\frac{z_1}{z_2}$$

Complex Division

$$\frac{z_1}{z_2} \cdot z_2 = z_1$$

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Therefore in polar coordinates,

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Therefore in polar coordinates,

$$|z_1/z_2| \;=\; |z_1|/|z_2| \qquad \text{and} \qquad \angle(z_1/z_2) \;=\; \angle z_1 \;-\; \angle z_2$$

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$$\frac{a+jb}{c+jd} \;=\; \frac{(a+jb)(c-jd)}{c^2+d^2} \;=\; \ldots$$

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- $c - jd \;=\; (c + jd)^*$ (complex conjugate)

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- $zz^* = |z|^2$

2. Which (one or more) of the following expressions equals

$$\frac{3+j}{j}\ ?$$

A. $10/(1+3j)$

B. $1+3j$

C. $1-3j$

D. $-(1+j)(1+2j)$

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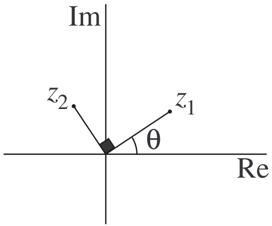
5. If z_1 and z_2 are as plotted below, where ■ denotes a right angle, which (one or more) of the following statements are correct?



A. z_2/z_1 is purely imaginary.

B. If $\theta = \pi/4$, then $z_1 z_2$ is real-valued.

C. If $\theta = \pi/4$, then z_1^*/z_2 is real-valued.

D. If $\theta = \pi/6$, then $(z_1)^4/z_2$ is real-valued.

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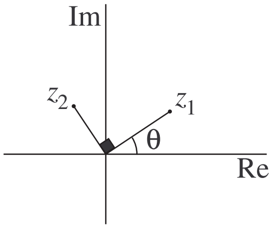

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Angles:

$$n(\angle z) = \angle v$$

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Each choice, $k = 0, 1, \ldots, n-1$, gives a different root.

6. (In Reverse!) Determine and plot the roots of

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Answer: