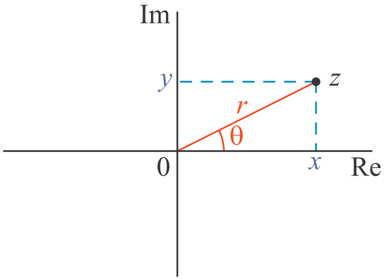
Cartesian and Polar Forms of a Complex Number



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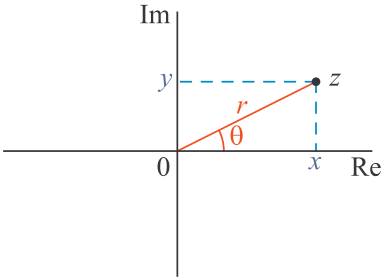

Cartesian coordinates:

$$x = \Re e\{z\} \; ; \qquad y = \Im m\{z\}$$

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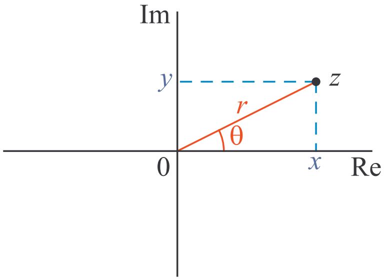


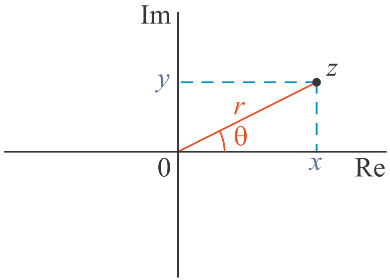
Cartesian coordinates:

$$x = \Re e\{z\} \; ; \qquad y = \Im m\{z\}$$

Polar coordinates:

$r = |z|$ (modulus or magnitude) ; $\theta = \angle z$ (angle)

Coordinate Conversions



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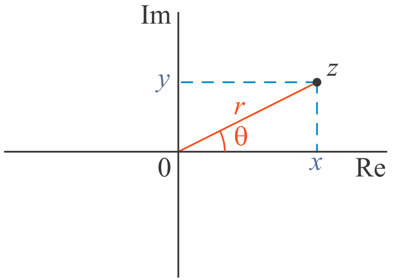
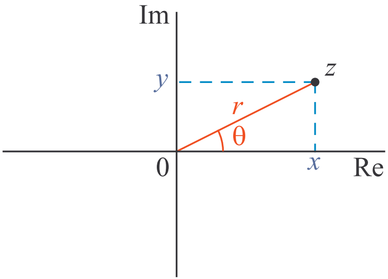


Polar to Cartesian:

$$x \;=\; r\cos\theta\,, \qquad y \;=\; r\sin\theta$$

Coordinate Conversions



Polar to Cartesian:

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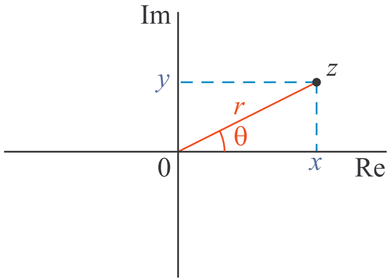
Cartesian to polar:

$$r = \sqrt{x^2 + y^2}$$

Coordinate Conversions



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$$r = \sqrt{x^2 + y^2}$$

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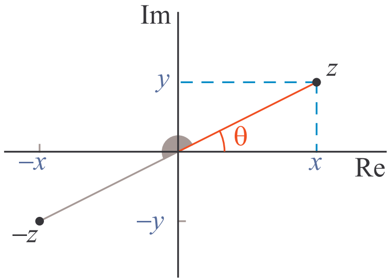
Coordinate Conversions



Polar to Cartesian:

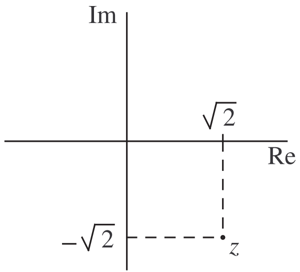
$$x = r\cos\theta\,, \qquad y = r\sin\theta$$

Cartesian to polar:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right) + \begin{cases} 0 & \text{if } x \geq 0 \\ \pi & \text{if } x < 0 \end{cases}$$

1. The complex number z is plotted below.



Its modulus and angle are given by

A. $|z| = \sqrt{2}$ and $\angle z = \pi/4$

B. $|z| = \sqrt{2}$ and $\angle z = -\pi/4$

C. $|z| = 2$ and $\angle z = -\pi/4$

D. $|z| = 2$ and $\angle z = -1$

Scaling and Addition

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Treat each z as a vector (from origin to z) and apply usual rules.

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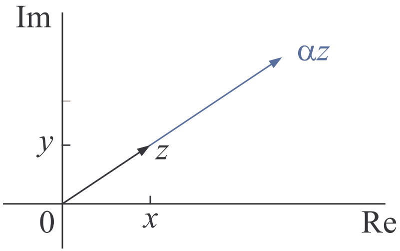

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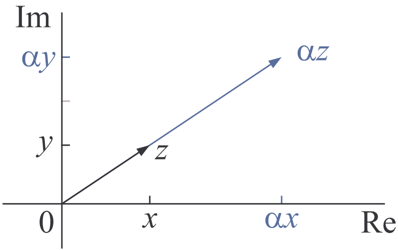

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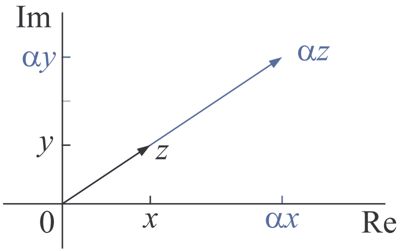

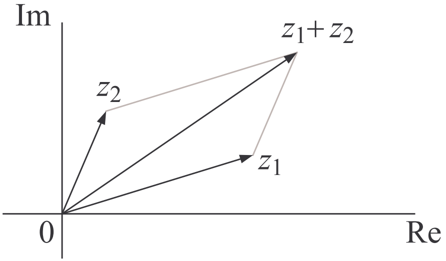

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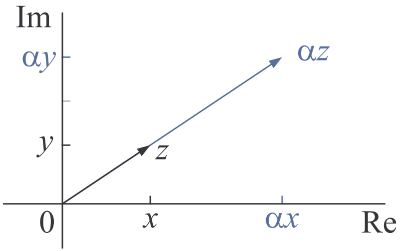
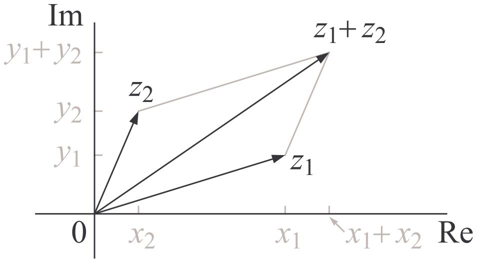




2. If the complex number z has modulus r and angle θ, which of the following is true about the complex number

$$w = -3z \ ?$$

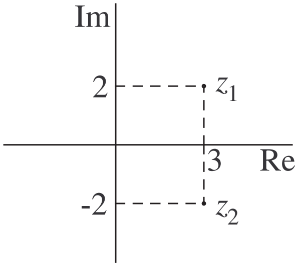
A. $|w| = 3|z|$ and $\angle w = -\theta$

B. $|w| = 3|z|$ and $\angle w = \theta + \pi$

C. $|w| = -3|z|$ and $\angle w = \theta$

D. $|w| = 9|z|$ and $\angle w = -\theta$

3. If z_1 and z_2 are as plotted below, what is the angle of the difference $z_1 - z_2$?



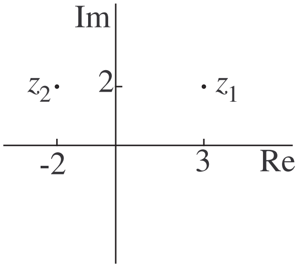
A. 0

B. $\pi/2$

C. π

D. $-\pi/2$

4. If z_1 and z_2 are as plotted below, what is value of $|z_1 + z_2|$?



A. $\sqrt{17}$

B. $\sqrt{21}$

C. $\sqrt{5}$

D. None of the above

The Equation $|z - z_0| = c$

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z_0 is fixed; z is variable

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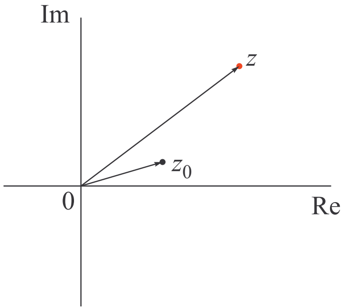

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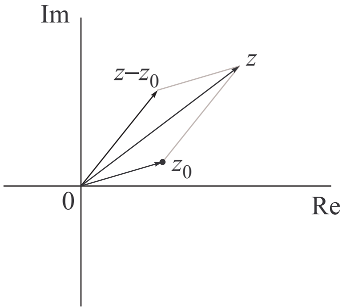

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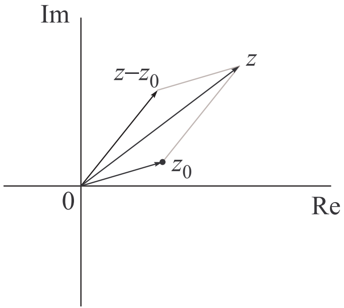

$|z - z_0| =$ distance of z from z_0

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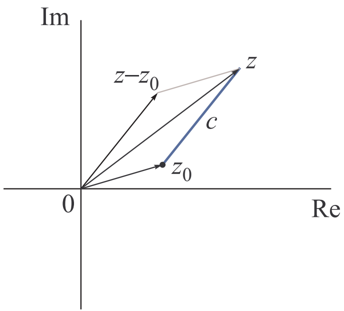

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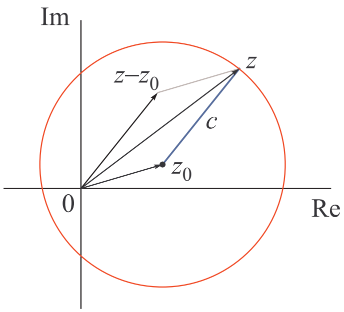

$|z - z_0| =$ distance of z from z_0

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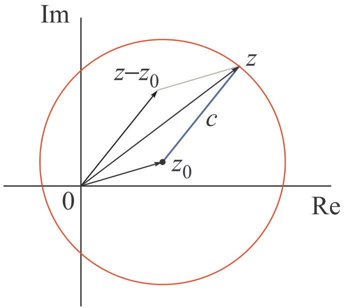
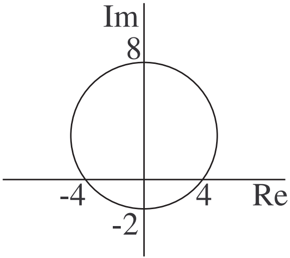


$|z - z_0| =$ distance of z from z_0

$|z - z_0| = c$:   circle of radius c centered at z_0

5. Which of the following equations describes the circle shown below?



A. $|z| = 4$

B. $|z| = 5$

C. $|z - 3| = 5$

D. $|z - 3j| = 5$

The Equation $|z - z_1| = |z - z_2|$

z_1 and z_2 are both fixed; z is variable

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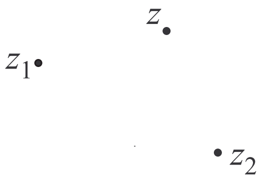

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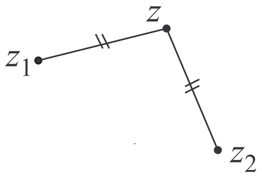

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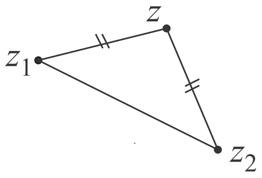

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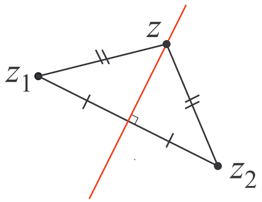

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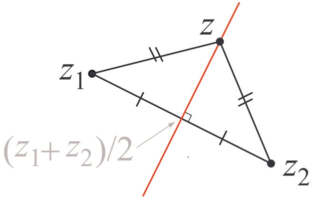

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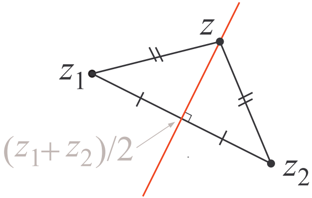
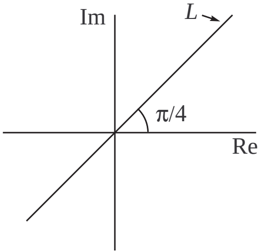


$|z - z_1| = |z - z_2|$: perpendicular bisector of the line segment joining z_1 and z_2

6. Which (one or more) of the following equations describe the line $\mathcal{L}$ shown below?



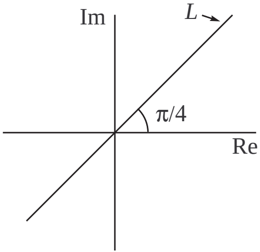
A. $|z-1| \;=\; |z-j|$

B. $|z-1| \;=\; |z+j|$

C. $|z-2| \;=\; |z-2j|$

D. $|z+1| \;=\; |z+j|$

6. Which (one or more) of the following equations describe the line $\mathcal{L}$ shown below?



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