All answers should be accompanied with proofs or sufficient explanation. Intermediate calculations should be shown.

In the problems you may use any representation of $F_{11}$ in the calculations. The final answers should be written using integers mod 11.

(a) (10pt). Prove that $\alpha = 2$ is a primitive element of $F = F_{11}$.

(b) (10pts). Let $C$ be a $[10, 6, d]$ RS code over $F$. Write out a parity-check $H$ matrix of $C$.

(c) (20pt) Reduce $H$ to a systematic form $H' = [I_4 | A]$.

Help: \[
\begin{pmatrix}
1 & 2 & 4 & 8
\end{pmatrix}^{-1} =
\begin{pmatrix}
2 & 1 & 8 & 1
\end{pmatrix}
(mod 11). (If you find no use of this equality, ignore it.)

(d) (30pt) Write out a generator matrix of $C$ in a systematic form
(Use caution: Lecture 2 applies to binary codes only)

(e) (10pt) Using $H'$, find the codeword $c_0$ that corresponds to the message symbols $(1, 1, 1, 1, 1)$.

(f) (10pt) What is the polynomial $f$ such that $\text{eval}(f) = c_0$?

(g) (15pt) Is it true that $(c_0, c_1, \ldots, c_9) \in C$ implies that $(c_9, c_1, c_2, \ldots, c_8) \in C$?

(h) (20pt) Let $c \in C$ be a vector of weight 5. Prove that if $c' \in C$ is such that $\text{supp}(c) = \text{supp}(c')$ then $c' = ac$ where $a \in F \setminus \{0\}$ is some constant.

(i) (15pt) Using problem 8, compute directly, with proof, the number of vectors of weight 5 in $C$ (your answer should be a number, not an expression).

(j) (30pt) Let $r = (3, 0, 0, 10, 5, 4, 0, 6, 10, 0)$ be a received vector. Perform the Peterson-Gorenstein Zierler algorithm to determine the number of errors and to decode the vector.

(k) (10pt) Explain how to find the polynomial $f$ such that $\text{eval}(f)$ equals the decoded codeword from problem 10.

(l) (10pt) If time left, invent a nice problem and solve it.