Problem 1.

The event \((A \cup B) \cap C\) is shown by bold line.

The events
\[
(A \setminus B) \cap C \quad (B \setminus A) \cap C \quad (A \cap B) \cap C
\]

are shown by the labels.

Since I, II and III are pairwise disjoint, we can use additivity and claim that
\[
P(I \cup II \cup III) = P(I) + P(II) + P(III) = P((A \cup B) \cap C)
\]

Problem 2: Consider all the possible outcomes of 2 rolls:

\[
P_Y(i) = \begin{cases} 
\frac{1}{36} & i = 1 \\
\frac{9}{36} & i = 2 \\
\frac{7}{36} & i = 3 \\
\frac{5}{36} & i = 4 \\
\frac{3}{36} & i = 5 \\
\frac{1}{36} & i = 6 
\end{cases}
\]
Prob. 3. The events $E_1$ and $E_2$ are equal. Indeed

$$A = \{1, 2, 3\}, \quad B = \{4, 5, 6\}$$

$$E_1 = (A \cap B)^C = (\{1, 3, 5\} \cap \{4, 5, 6\})^C = \{1, 2, 3, 4, 6\}$$

$$E_2 = (A^C \cup B^C) \cup (A \cup B)^C \cup (A^C \cup B)^C$$

$$= (\{1, 3, 5\} \cup \{1, 2, 3\})^C \cup (\{1, 3, 5\} \cup \{4, 5, 6\})^C$$

$$\cup (\{2, 4, 6\} \cup \{4, 5, 6\})^C$$

$$= \{4, 6\} \cup \{2\} \cup \{1, 3\}$$

$$= E_1$$

Venn diagram

\[
\begin{align*}
(A \cup B)^C &= A^C \cap B^C \\
(A^C \cup B)^C &= A^C \cap B \\
(A^C \cup B^C)^C &= A \cap B
\end{align*}
\]

Prob. max $P(A \cap B) = 0.5$ is attained when $A \subset B$.

min $P(A \cap B) = 0.1$ is attained when $\Omega = B \cup (A \setminus B)$

Prob. 5 Let $X_1 Y_1, X_2 Y_2, X_3 Y_3$ ... be the sequence of throws in this game.

The event that Player 1 wins is composed of the following outcomes:

$W = \{H\} \cup \{TTTH\} \cup \{TTTTH\} \cup ... \cup \{(2k)T, H\}$

$P(W) = \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + ... + \frac{1}{2^{2k+1}} = \frac{1}{2} \sum_{k=0}^{\infty} \frac{1}{2^{2k}} = \frac{1}{2} \cdot \frac{1}{3/4} = \frac{2}{3}$
Problem 6: 

\[ P(A) = \frac{2}{3} \]

\[ P(B) = \frac{1}{3} \]

\[ P(C) = \frac{1}{2} = \int_{0}^{1} \left( \int_{0}^{x} dy \right) dx \]

\[ P(C|A) = \int_{\frac{1}{3}}^{1} \int_{0}^{x} dy \, dx = \left. \frac{x^2}{2} \right|_{\frac{1}{3}}^{1} = \frac{1}{2} - \frac{1}{18} = \frac{4}{9} \]

(1) \( P(AB) = P(A)P(B) \) because the points \( x \) and \( y \) are chosen independently. Thus \( A \) and \( B \) are independent.

(2) \( P(AC) = P(A)P(C|A) = \frac{2}{3} \cdot \frac{4}{9} = \frac{8}{27} \neq P(A)P(C) = \frac{1}{3} \)

Thus \( A \) and \( C \) are dependent.