ENEE324. Problem set 5

Date due March 30, 2016

Explanations are required, no credit for just the answer.

1. Consider the function \( \phi(x) = 0 \) for \( x < 1 \) or \( x > 5 \) and

\[
\phi(x) = \frac{1}{2} - \frac{1}{4}|x - 3|, \quad 1 \leq x \leq 5.
\]

Plot the graph of \( \phi(x) \). Does it represent a valid pdf? If yes, compute the cdf (make sure you give the answer for all \( x, -\infty < x < \infty \)).

2. Let \( f_X(x) = 2x\pi^{-2}, 0 < x < \pi \). Define the RVs \( Y = \sin(X), Z = \sin(X/2) \). Find \( f_Y(y), f_Z(z) \) (you can use the method described on p.202 of the textbook).

3. A snowstorm continues for 1 hour, and the accumulation of snow on the ground in a particular spot is described by an RV \( X \) with pmf \( f_X(x) = 6x(1-x), 0 < x < 1 \) and \( f_X(x) = 0 \) otherwise. What’s the probability that the amount of snow at the end of the storm is within two standard deviations of the mean?

4. Packets arrive at a server one by one with exponentially distributed time between arrivals, \( f_T(t) = \lambda e^{-\lambda t}, t \geq 0 \). (a) What is the probability that there are no arrivals in the next \( t \) minutes? (b) What is the probability that the next packet arrives within \( t \) minutes, \( a \leq t \leq b \), where \( a, b \) are some given numbers?

5. We choose a random point \( X \) in the segment \([0, \pi/4]\). Find \( E(\cos 2X), E(\cos^2(X)) \).

6. A random point \( X \) is chosen from the interval \((0, 1)\). What is the distribution of the RV \( Y = 5X - 1 \)?