ENEE324-03. Problem set 1

Date due February 10, 2016

1. Let $A$ be the event that on a given day the stock market goes up, $B$ the event that the dollar rate against the euro becomes higher, and let $C$ be the event that there is a coup-d'état in Boomerangia. The event that $A$ and $B$ but not $C$ happen on some fixed day is expressed as $A \cap B \cap C^c$. Write similar formulas for the events
(i) only $A$ occurs 
(ii) at least one of the three described events occurs 
(iii) at least two events of the events occur 
(iv) all three events occur 
(v) none of the events occur 
(vi) at most two of the events occur 
(vii) exactly two of the events occur.  
Use the union, intersection, complement, and set difference operations.

2. An absent-minded driver leaves home without checking his car. With probability 0.4 he will need a spare tire, with probability 0.2 he’ll run out of gas, and with probability 0.15 both occur. Find the probability that he’ll have no troubles on his trip. (Use Venn diagrams)

3. The same driver uses a different car. A spare tire will be required with probability 0.7, and he’ll run out of gas with probability 0.8, but with this car he doesn’t know what’s the probability of both troubles happening together. Show that the probability that he’ll have {a flat} \cap {no gas} is no less than 0.5.

4. A snowbank on campus always melts away in 2 weeks. Suppose that the probability that it melts in $k$ or fewer days is $k/14$ for all $k = 1, 2, \ldots, 14$. Given that the snowbank is still there after one week, what’s the probability that it will take all 14 days to melt?

5. An urn contains 12 red and 6 blue balls. 3 balls are drawn without replacement (i.e., once the ball is taken out, we do not return it to the urn). Each time each of the balls is drawn with equal probability among the remaining balls.
   (a) Find the probability that all three balls are red.
   (b) Find the probability that exactly one of the three balls is blue.

6. A car is parked in a restricted spot, and the enforcement draws chalkmarks on the wheel to see if the car has moved after a certain amount of time. After some time, the wheel (assume that it is a circle of unit circumference), has 3 disjoint arcs of length 1/3, 1/2 and 1/6 marked on it. The owner drives home, and is waiting at a traffic light. The arc of length 1/6 does not touch the ground. What is the probability that the arc of length 1/3 is touching the ground? (the arc is touching the ground if the tangency point of the wheel and the ground is somewhere within the arc).