

PROBABILITY — Ph.D. Qualifying Exam Spring 2009**(i) (7 pts.)**

Three identical (in exterior appearance) urns contain four balls each. Two of the urns contain an equal number (two) of red and green balls, while the third urn contains three red balls and one green ball.

An urn is chosen at random, and two balls are taken out of it without replacement. Given that both balls are red, what is the conditional probability that the chosen urn contained two red and two green balls?

(ii) (6 pts.)

A discrete random variable Z takes values -1 , 0 and $+1$ with probabilities $(1 - c)/3$, $1/3$ and $(1 + c)/3$, respectively, where $-1 \leq c \leq 1$. Determine $\text{Var}[Z]$ (the variance of Z), as well as the values of c which result in the minimum and maximum values of $\text{Var}[Z]$.

(iii) (7 pts.)

X is uniformly distributed over the interval $[0, 1]$. For every $x \in [0, 1]$, the conditional distribution of Y given $X = x$ is uniform over the interval $[0, x^2]$. Determine the probability that Y is greater than $1/4$.