

PROBABILITY — Ph.D. Qualifying Exam Fall 2009

(i) (7 pts.)

Each day at midnight, the outgoing telephone call traffic of a certain organization is switched to one of two networks; this routing is maintained for exactly 24 hours. The probability of Network 1 (respectively, Network 2) carrying the traffic for the day is α (respectively, $1 - \alpha$). The probability that a call is dropped by Network 1 (respectively, Network 2) is p (respectively q); it can be assumed that different calls are completed or dropped independently of one another.

Suppose that on a certain day, the first dropped call is the n^{th} call made that day. Given this event, what is the conditional probability that Network 1 has been carrying the traffic for that day?

(ii) (6 pts.)

The density of a random variable T is given by

$$f_T(t) = ct^2, \quad 0 \leq t \leq b$$

and is equal to zero for $t \notin [0, b]$. If $E[T] = 3$, what is the value of $\text{Var}[T]$?

(iii) (7 pts.)

Let X and Y be independent variables, each uniformly distributed over $[0, 1]$. If A is the event that $X + Y \geq 3/2$, determine $P(X \leq x | A)$ for every $x \in [0, 1]$, as well as the conditional density $f_X(x|A)$.