LINEAR SYSTEMS AND SIGNALS — Ph.D. Qualifying Exam Fall 2009

(i) (7 pts.)
For each of the two signals

$$x_1(t) \;=\; e^{-2t}u(t) + e^{-t}u(-t) \qquad \text{and} \qquad x_2(t) \;=\; e^{-t}u(t) + e^{-2t}u(-t),$$

determine the region of convergence of the respective Laplace transform.

(ii) (6 pts.)
Consider the discrete-time sinusoidal signals

$$x_1[n] \;=\; \sin(\hat{\omega} n) \qquad \text{and} \qquad x_2[n] \;=\; \sqrt{3}\cos(\hat{\omega} n)$$

where $\hat{\omega}$ is in radians and n ranges over all integers.

- **(2 pts.)** Specify the condition on $\hat{\omega}$ under which the signals are periodic in n.
- **(4 pts.)** Express
$$y[n] \;=\; x_1[n] + x_2[n]$$
in the form $A\cos(\hat{\omega} n + \phi)$.

(iii) (7 pts.)
The standard Gaussian pulse

$$g(t) \;=\; \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}$$

has total area (i.e., $\int_{-\infty}^{\infty} g(t)\, dt$) equal to 1.
Derive the Fourier transform $G(\omega)$ of $g(t)$ by the method of your choice. (*Hint*: Use

$$\begin{array}{rcll} x'(t) & \longleftrightarrow & j\omega X(\omega) & \text{(Fourier transform of a derivative)} \\ -jtx(t) & \longleftrightarrow & X'(\omega) & \text{(derivative of a Fourier transform)} \\ g'(t) & = & -tg(t) & \text{(prove for the Gaussian pulse)} \end{array}$$

to obtain a relationship between $G'(\omega)$ and $\omega G(\omega)$.)