

BASIC MATHEMATICS – Ph.D. Qualifying Exam Spring 2009

Important: In all cases, you must show your work to receive any credit.

(i) (6 pts.)

Calculate

$$\lim_{x \rightarrow 0} \left(\frac{1}{\ln(x+1)} - \frac{1}{x} \right)$$

(ii) (7 pts.)

Solve the differential equation

$$\frac{dx}{dt} + x = e^{-2t}$$

subject to the initial condition $x(0) = 2$.

(iii) (7 pts.)

Consider the following two vectors in 3-dimensional space:

$$\mathbf{u} = \hat{\mathbf{x}} + 2\hat{\mathbf{y}} + 2\hat{\mathbf{z}} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \hat{\mathbf{x}} + \hat{\mathbf{z}} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Find a vector $\hat{\mathbf{w}}$ with unit length that is perpendicular to \mathbf{u} and lies in the plane spanned by \mathbf{u} and \mathbf{v} .