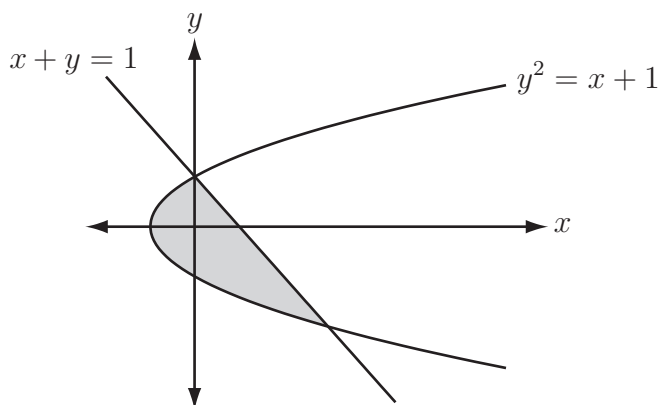


BASIC MATHEMATICS – Ph.D. Qualifying Exam Fall 2009

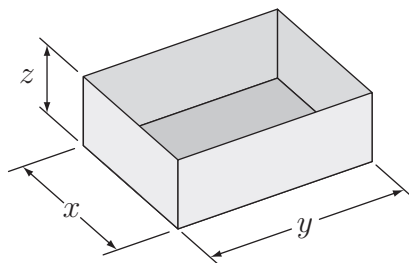
(i) (6 pts.)

Find the area enclosed by the two curves $y^2 = x + 1$ and $x + y = 1$, shown below.



(ii) (7 pts.)

Consider a rectangular cardboard box that is **open** on the top, as shown below:



The total volume of the box is constrained to be 4 cubic inches (i.e., $xyz = 4$.) How should you choose the dimensions x , y and z in order to minimize the amount of cardboard needed to construct the box?

(iii) (7 pts.)

Find a solution $y(t)$ to the differential equation

$$\frac{d^2y}{dt^2} + y = t^2$$

of the form $y(t) = at^2 + bt + c$.

Then find a solution to the same differential equation that satisfies the initial conditions

$$y(0) = \dot{y}(0) = 0$$