

PROBABILITY—Ph.D. Qualifying Exam Spring 2008

**Part A (6 pts.)**

Information source  $\mathcal{X}$  produces a sequence of independent bits  $X_i$  such that  $P(X_i = 1) = 3/5$  and  $P(X_i = 0) = 2/5$  for all  $i$ . Independently of  $\mathcal{X}$ , information source  $\mathcal{Y}$  also produces a sequence of independent bits  $Y_i$  such that  $P(Y_i = 1) = 1/3$  and  $P(Y_i = 0) = 2/3$  for all  $i$ .

Compute the probability that the first eight bits produced by  $\mathcal{X}$  are identical to the first eight bits produced by  $\mathcal{Y}$ .

**Part B (7 pts.)**

A capacitor-resistor circuit is switched on at a random time  $T$ . The voltage across the resistor varies as

$$v(t) = \begin{cases} a2^{-(t-T)}, & t \geq T; \\ 0, & t < T, \end{cases}$$

where  $a$  is the nonrandom source voltage.

If  $T$  is uniformly distributed over the time interval  $[0, 4]$ , what is the probability that  $v(3)$  exceeds  $a/2$ ?

**Part C (7 pts.)**

Let  $(X, Y)$  be the Cartesian coordinates of a point which is uniformly distributed over the interior of the unit circle. Determine  $E[X^2 + Y^2]$ .