

## LINEAR SYSTEMS—Ph.D. Qualifying Exam Fall 2008

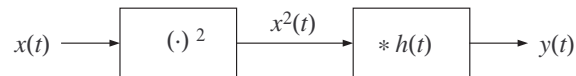
### Part A (8 pts.)

Consider the continuous-time linear time-invariant (LTI) system with impulse response

$$h(t) = \begin{cases} e^{-2t}, & t \geq 0; \\ 0, & t < 0. \end{cases}$$

(A1) (3 pts.) Determine the complex frequency response  $H(j\omega)$ , where  $\omega$  is in radians per unit time.

(A2) (5 pts.) Let  $x(t) = \cos(6\pi t)$  be the input to a cascade consisting of a squarer followed by the LTI system given above. Determine the output  $y(t)$  of the cascade.



### Part B (6 pts.)

Suppose that the signal  $x_1(t)$  has bandwidth  $B_1$  Hz, i.e., its spectrum vanishes for frequencies outside the range  $(-B_1, B_1)$  Hz. Similarly, the signal  $x_2(t)$  has bandwidth  $B_2$  Hz. What are the bandwidths  $B_3$  and  $B_4$  of the signals defined by

- $x_3(t) = x_1(t)x_2(t)$
- $x_4(t) = (x_1 * x_2)(t)$  (i.e., convolution)

*Justify your answers.*

### Part C (6 pts.)

Consider the discrete-time filter whose input  $x[n]$  and output  $y[n]$  are related by

$$y[n] = x[n] + 3x[n-1] + ax[n-2] + bx[n-3],$$

where  $a$  and  $b$  are fixed (time-invariant) coefficients. Determine the values of  $a$  and  $b$  such that every periodic input  $x[n]$  with period 4 produces a periodic output  $y[n]$  with period 2.



